UNIT I

RANDOM VARIABLES

PART – A

1. If Var(x) = 4, find Var (3x+8), where X is a random variable.
   Solution: Var (ax+b) = a^2 Var x
   Var (3x+8) = 3^2 Var x = 36

2. If a random variable X takes the values 1,2,3,4 such that 2P(X=1) = 3P(X=2) = P(X=3) = 5P(x=4). Find the probability distribution of X.
   Solution: Let P(X=3)= k,
   P(X=1) = k/2
   P(X=2) = k/3
   P(X=4) = k/5
   $$\frac{k}{2}+\frac{k}{3}+k+\frac{k}{5}=1$$
   $$\Rightarrow \quad k=\frac{30}{61}$$
   P(X=1) = 15/61
   P(X=2) = 10/61
   P(X=3)= 30/61
   P(X=4)= 6/61
3. A continuous random variable X has probability density function given by f(x) = $3x^2$ 0,x,1. Find K such that P(X>K) = 0.05.
   Solution:
   $$P(X \le K) = 0.95$$
   $$\int_0^K 3x^2\,dx = 0.5 \qquad \Rightarrow K^3 = 0.95$$
   $$K = (0.95)^{\frac{1}{3}} = 0.983$$
4. Find the cumulative distribution fuction F(x) corresponding to the p.d.f. f(x) = $\dfrac{1}{\pi(1+x^2)}$,$-\infty < x < \infty$

Solution:

$$F(x) = \int_{-\infty}^{x} \frac{1}{\pi(1+x^2)}\,dx$$

$$= \frac{1}{\pi}\left[\tan^{-1} x\right]_{-\infty}^{x}$$

$$= \frac{1}{\pi}\tan^{-1} x + \frac{1}{2}$$

5. If a RV X has the moment generating function $M_x(t) = \dfrac{2}{2-t}$

   Determine the variance of X.

   Solution:

$$M_x(t) = \frac{2}{2-t} = \left(1 - \frac{t}{2}\right)^{-1}$$

$$E(X) = \frac{1}{2} \quad E(X^2) = \frac{1}{2}$$

$$Var(x) = E(X^2) - (E(X))^2 = \frac{1}{4}$$

6. In a binomial distribution the mean is 4 and variance is 3, Find P(X=0).

   Solution: np=4, npq=3

   Hence q=3/4, p=1-q=1/4,

   Since np=4 , n=16.

$$P(X = x) = nc_x p^x q^{n-x}$$

$$= 16c_x\left(\frac{1}{4}\right)^x\left(\frac{3}{4}\right)^{16-x} \quad x = 0,1,2,\ldots$$

$$P9\ X = 0) = \left(\frac{3}{4}\right)^{16} = 0.01002$$

7. The moment generating function of a random variable X is given by $M_x(t) = e^{3(e^t-1)}$. Find P(X=1).

   Solution: $\lambda = 3$

$$f(x) = \frac{e^{-3}3^x}{x!},\ x = 1,2,3,\ldots$$

$$P(X = 1) = \frac{e^{-3}3^1}{1!} = 0.1494$$

8. Find the moment generating function of uniform distribution.

$$M_x(t) = \int_{-\infty}^{\infty} e^{tx} f(x)dx$$

Solution: $$= \int_a^b e^{tx} \frac{1}{b-a} dx$$

$$= \frac{e^{bt} - e^{at}}{t(b-a)}$$

9. What are the properties of Normal distribution?

Solution:

- The normal curve is symmetrical when p=q or p≈q
- The normal curve is a single peaked curve
- The normal curve is asymptotic to x-axis as y decreases rapidly when x increases numerically.
- The mean, median and mode coincide and lower and upper quartiles are equidistant from the median
- The curve is completely specified by mean and standard deviation along with the value of y_o

10. The life time of a component measured in hours is Weibull distribution with parameter $\alpha = 0.2,\ \beta = 0.5.$ Find the mean lifeti;me of the component.

Solution:

Mean = E(X)= $\alpha^{-1/\beta}\Gamma\left(1+\frac{1}{\beta}\right)$

The mean life of the component = $0.2^{-1/0.5}\Gamma\left(1+\frac{1}{0.5}\right) = 50\ hours$

11. If X is binomially distributed with n=6 such that P(X=2)=9P(X=4), find E(x) and Var(x).

Solution: $6C_2\ p^2q^4 = 9\ (6C_4\ p^4q^2)$; q=3p ;p=1/4.

E(X)=1.5 ; Var(X) = 9/8

13. If f(x) = kx^2 , 0<x<3, is to be a density function, find the value of k.

Solution: $\int_0^3 kx^2\,dx = 1; 9k = 1; k = \frac{1}{9}$

PART – B

Refer: (A) Probability and statistics by M. Sundravalli and R. Mahadevan

(B) Probability and statistics by T. Veerarajan.

(C) Probability Queueing Theory by G. Balaji.

1. Find the mean and variance of the following distributions: Binomial, Poisson(Refer B: page no:179,183)

2. Find the mean and variance of the following distributions : Geometric and Exponential(Refer B : page no:185,212)

3. Prove that Poisson distribution is the limiting form of Binomial distribution. (Refer B: page no : 181)

4. For a triangular distribution $f(x) = \begin{cases} x, & 0 < x \le 1 \\ 2 - x & 1 \le x \le 2 \\ 0 & otherwise \end{cases}$

Find the mean, variance and moment generating function. (Refer C: page no : 1.122)

5.It is known that the probability of an item produced by a certain machine will be defective is 0.05. If the produced items are sent to the market in packets of 20, fine the no. of packets containing at least, exactly and atmost 2 defective items in a consignment of 1000 packets using (i) Binomial distribution (ii) Poisson approximation to binomial distribution.

(Refer C: page no : 2.28)

6. The daily consumption of milk in excess of 20,000 gallons is approximately exponentially distributed with $\theta = 3000$. The city has a daily stock of 35,000 gallons. What is the probability that of two days selected at random, the stock is insufficient for both days.
(Refer C: page no : 2.74)

7. Each of the 6 tubes of a radio set has a life length (in yrs) which may be considered as a RV that follows a weibull distribution with parameters $\alpha = 25$ *and* $\beta = 2$. If these tubes function independently of one another,

what is the probability that no tube will have to be replaced during the first 2 months of service?
(Refer C: page no : 2.82)

UNIT II

TWO- DIMENSIONAL RANDOM VARIABLES

PART – A

1. State Central limit theorem.
   Solution: If X is the mean of a sample of size n taken from a population having the mean and the finite variance σ^2, then

$Z = \dfrac{X-\mu}{\sigma/\sqrt{n}}$ is a random variable whose distribution function approaches that of the standard normal distributions as n tends to infinity.

2. Define discrete probability Distribution.

   Solution: The mathematical definition of a discrete probability function, p(x) is a function that satisfies the following properties.

   a) The probability that x can take a specific value is p(x). That is $P(X=x)=p(x)=p_x$
   b) P(x) is non-negative for all real x.
   c) The sum of p(x) over all possible values of x is 1, that is $\sum p(j) = 1$ where j represents all possible values that x can have and p_j is the probability at x_j.

3. What is conditional probability?

   Solution: The conditional probability of X given Y is $f(y/x)= f(x,y)/f_2(y)$.

4. Define Marginal distribution function.

   Solution: $P(X \le x) = F_1(x) = \int_{u=-\infty}^{x} \int_{v=-\infty}^{\infty} f(u,v)\,du\,dv$

   $P(Y \le y) = F_2(x) = \int_{u=-\infty}^{\infty} \int_{v=-\infty}^{y} f(u,v)\,du\,dv$

   The above eqns are called the marginal distribution functions or distribution functions of X and Y.

5. Define Marginal density function.

   Solution: $P(X \le x) = F_1(x) = \int_{u=-\infty}^{x} \int_{v=-\infty}^{\infty} f(u,v)\,du\,dv$

   $P(Y \le y) = F_2(x) = \int_{u=-\infty}^{\infty} \int_{v=-\infty}^{y} f(u,v)\,du\,dv$

   The derivatives of the above equations with respect to x and y are then called the marginal density functions or simply the density functions, of X and Y and are given by $f_1(x)= \int_{v=-\infty}^{\infty} f(x,y)\,dv$ $f_2(y)= \int_{u=-\infty}^{\infty} f(x,y)\,du$

6. What is Covariance?

Solution: If X and Y are two random variables with the respective expected values $\overline{X}$, $\overline{Y}$ then the covariance between X and Y is defined by the relation μ_{11}=Cov(X,Y)=E((X-$\overline{X}$)(Y-$\overline{Y}$))

7. Define regression.
Solution: The main purpose of curve fitting is to estimate one of the variables (the dependent variable) from the other(the independent variable). The process of estimation is often referred to as regression.

8. What is a scatter diagram?
Solution: A scatter diagram is a graphical representation of data points for a particular sample. Choosing a different sample, or enlarging the original one can obtain a different scatter diagram.

9. If X represents the total number of heads obtained, when a fair coin is tossed 5 times, find the probability distribution of X.
Soln.:

X	0	1	2	3	4	5
P_x	1/32	5/32	10/32	10/32	5/32	1/32

10. If the probability distribution of X is given as:

x	1	2	3	4
P_x	0.4	0.3	0.2	0.1

Find $P\left(\frac{1}{2}<X<\frac{7}{2} / X>1\right)$.

Solution: $\dfrac{P\left\{\left(\frac{1}{2}<X<\frac{7}{2}\right)\cap(X>1)\right\}}{P(X>1)} = \dfrac{P(X=2or3)}{P(X=2,3or4)} = \dfrac{0.5}{0.6} = \dfrac{5}{6}$

11. If the pdf of X is f(x)=2x , 0<x<1, find the pdf of (i)Y=3X+1.
(ii) y=$\sqrt{X}$

Solution: (i) f(y) = $\left|\frac{dx}{dy}\right| f(x) = \frac{2}{9}(y-1) in 1<y<4$

(ii)f(y) = 2y x 2y^2 = 4y^3 in 0 <y <1.

12. If the RV X is uniformly distributed in (0,2), find the pdf of

Y= X^3.

Solution: $f(y) = \left|\frac{1}{2} y^{-\frac{2}{3}}\right| \frac{1}{2} = \frac{1}{6} y^{-\frac{2}{3}}$ $0 < y < 8$.

13. If the R.V X is uniformly distributed in (-1,1) , find the pdf of Y = |X|.

Solution: $f(y) = \frac{1}{2} + \frac{1}{2} = 1$ in $0 < y < 1$.

14. If the joint pdf of (X,Y) is given by f(x,y)=2 in $0 \le x < y \le 1$. Find E(x).

Solution: $E(x) = \int_0^1 \int_0^y 2x\,dx\,dy = \int_0^1 y^2\,dy = \frac{1}{3}$

15. When are 2 random variables orthogonal?

Solution: If E(XY)= 0.

PART – B

Refer: (A) Probability and statistics by M. Sundravalli and R. Mahadevan

(B) Probability and statistics by T. Veerarajan.

(C) Probability Queueing Theory by G. Balaji.

1. State and prove the central limit theorem for independent and identically distributed random variables.
   Solution: Refer(B) page no: 165.

2. (a) If X_1 and X_2 are independent Poisson variates with parameters λ_1 and λ_2, find the conditional distribution of X_1 for a given X_1+X_2.

   (b). The joint density function of 2 continuous random variables X and Y is
   $$f(x,y) = \begin{cases} c\,xy, & 0<x<4,\ 1<y<5 \\ 0 & \text{otherwise} \end{cases}$$

   1. Find $P[X \ge 3, Y \le 2]$
   2. Find marginal distribution function of X.

   Solution: Refer (A) page no: 81-86

3. The joint density function of a RV (X,Y) is
   $f(x,y)= 8xy$, $0 < x < 1$; $0 < y < x$;
   Find the marginal density functions.
   Find the conditional density function f(y/x).
   Solution: Refer (B) page no: 81(prob 34).

4. The joint pdf of the RV (X,Y) is given by
   $f(x,y) = kxy\, e^{-(x^2+y^2)}$, $x>0$, $y>0$
   Find the value of k and prove also that X and Y are independent.
   Solution: Refer (B) page no: 73.

5. The joint probability mass function of (X,Y) is given by $p(x,y) = k(2x + 3y)$; $x=0,1,2$; $y=1,2,3$.
   Find all the marginal and conditional probability distributions. Also find the probability distribution of (X+Y).
   Solution: Refer (B) page no:65.

6. If X and Y are independent RVs each following N(0,2),
   Find the pdf of Z=2X+3Y.
   Solution: Refer (B) : page no :106.

7. If X and Y each follow an exponential distribution with parameter 1 and are independent , find the pdf of U=X-Y.
   Solution: Refer (B) : page no : 107.

8. If the joint pdf of (X,Y) is given by $f(x,y) = x+y$; $0 \le x,y \le 1$, find the pdf of U=XY.
   Solution: Refer (B) : page no : 108.
9. If X and Y are independent RVs with $f(x) = e^{-x}\, U(x)$ and $f(y)= 3e^{-3y}\, U(y)$, find f(z) if Z=X/Y
   Solution: Refer (B): page no:109.

10. If X,Y and Z are uncorrelated RVs with zero means and standard deviations 5,12 and 9 respectively and if U=X+Y and V=Y+Z, find the correlation coefficient between U and V.
    Solution: Refer B : page no: 132

11. If the joint pdf of (X,Y) is given by $f(x,y) = x + y,\ 0 \le x,\ y \le 1$, find ρ_{xy}.
    Solution: Refer B: page no:140 (prob 64).

12. The lifetime of a certain brand of an electric bulb may be considered a RV with mean 1200 h and standard deviation 250h. Find the probability, using central limit theorem, that the average lifetime of 60 bulbs exceeds 1250 h.
    Solution: Refer B: page no : 165

13. A distribution with unknown mean has variance equal to 1.5. Use Central limit theorem to find how large a sample should be taken from the distribution in order that the probability will be atleast 0.95 that the sample mean will be within 0.5 of the population mean.
    Solution: Refer B: page no :166

14. If $X_1, X_2, \ldots, X_n$ are Poisson variates with parameter $\lambda=2$, use central limit theorem to estimate $P(120 \le S_n \le 160)$, where $S_n = X1+X2+\ldots+X_n$ and n=75.
    Solution: Refer B: page no: 167.

15. The joint pdf of a two dimensional RV (X,Y) is given by $f(x,y)= xy^2 + x^2/8,\ 0 \le x \le 2,\ 0 \le y \le 1$. Compute $P(X \ge 1)$, $P(Y< ½)$, $P(x >1/y < 1/2)$, $P(X < Y)$
    Solution: REFER (B) , pg no : 69

UNIT III

MARKOV PROCESSES AND MARKOV CHAINS

REFERENCE : "PROBABILITY, STATISTICS & RANDOM PROCESS " - T. VEERARAJAN

PART A

1. Define Random process. (Pg – 337)
   Solution: A random process is a collection of random variables {X(s,t)} that are functions of a real variable, namely t where

s∈ (sample space) and t∈T(parameter set).

2. Give the classification of Random Processes.(Pg – 338)
   Solution: Discrete random sequence, Continuous random sequence, Discrete random process, Continuous random process.

3. Define Stationary processes. (Pg – 339)
   Solution: If certain probability distribution or averages do not depend on t, then the random process {X(t)} is called stationary.

4. Define SSS process.(Pg – 340)
   Solution: A random process is called a strongly stationary process or strict sense stationary process, if all its finite dimensional distributions are invariant under translation of time parameter.

5. Define WSS process.(Pg – 341)
   Solution: A random process {X(t)} with finite first and second order moments is called a weakly stationary process or covariance stationary process or wide-sense stationary process, if its mean is a constant and the auto correlation depends only on the time difference.

6. Is Poisson process covariance stationary? Justify.(Pg – 343)
   Solution: No. Mean of poisson process = $\lambda t \neq$ a constant.

7. Show that the random process $X(t)=A\cos(w_0t + \theta)$ is WSS, if A and w_0 are constants and θ is a uniformly distributed RV in $(0 , 2\pi)$. (Pg – 344)
   Solution: Mean = 0 = a constant.
   Autocorrelation = $(A^2/2)\cos w_0 (t_1 - t_2)$.
   Hence WSS process.

8. If {X(t)} is a wss process with autocorrelation $R(\tau)=Ae^{-\alpha|\tau|}$, determine the second-order moment of the RV X(8) – X(5).
   Solution: $2A(1 - e^{-3\alpha})$

9. Define Markov process. (Pg – 446)
   Solution: Markov process is one in which the future value is independent of the past values, given the present value

10. Define Markov chain.(Pg – 447).
Solution: Random processes with Markov property which takes discrete values, whether t is discrete or continuous, are called Markov chains.

11. Prove that the matrix $\begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1/2 & 1/2 & 0 \end{pmatrix}$ is the tpm of the irreducible Markov chain. (Pg – 458 ;eg: 7)

12. . State Chapman-Kolmogorov theorem.(Pg – 448)
Solution: If P is the tpm of a homogeneous Markov chain, then the n-step tpm $P^{(n)} = P^n$.

13. If the tpm of a Markov chain is $\begin{pmatrix} 0 & 1 \\ 1/2 & 1/2 \end{pmatrix}$
find the steady-state distribution of the chain.(Pg – 460)
Sol: 1/3 , 2/3

PART B

1(a). Define Poisson Process.(Pg – 434)
Derive the Probability Law for the Poisson Process. (Pg – 435).
(b). What is the mean and autocorrelation of the Poisson Process?(Pg – 436)

2(a). Prove that the interarrival time of a Poisson process with parameter λ has an exponential distribution with mean $(1 / \lambda)$. (Pg – 438)
(b). If the customers arrive at a bank according to a Poisson process with mean rate of 2 per minute, find the

probability that, during a 1 – min interval, no customer arrives.(Pg 444)

3. A radioactive source emits particles at rate of 5 per minute in accordance with Poisson process. Each particle emitted has a probability 0.6 of being recorded. Find the probability that 10 particles are recorded in 4 – min period. (Pg – 442)

4. Patients arrive randomly and independently at a doctor's consulting room from 8 A.M. at an average rate of one in 5 min. The waiting room can hold 12 persons. What is the probability that the room will be full when the doctor arrives at 9 A.M.?(Pg – 445)

5. Suppose that the customers arrive at a counter independently from 2 different sources. Arrivals occur in accordance with a Poisson process with mean rate of 6 per hour from the first source and 4 per hour from the second source. Find the mean interval between any 2 successives arrivals.(Pg – 445)

6. Define Autocorrelation and Autocovariance of the Random process.(Pg – 340)
Define Correlation coefficient, Cross-correlation,cross-covariance of Random process. (Pg – 340).
Define Random walk.(Pg-350)
Define a semirandom telegraph signal process and random telegraph signal process . Are they stationary? (Pg -353)

7. State and prove the additive property of Poisson process. (Pg – 437)
Prove that the difference of 2 independent Poisson Process is not a Poisson Process. (Pg – 438)

8. A man either drives a car or catches a train to go to office each day. He never goes 2 days in a row by train but if he drives one day, then the next day he is just as likely to drive again as he is to travel by train. Now suppose that on the first day of the week, the man tossed a fair dice and drove to work if and only if a 6 appeared. Find (a) the probability that he takes a train on the third day and (b) the probability he drives to work in the long run.(Pg –453 ; eg:3)

9. A gambler has Rs. 2/- . He bets Rs.1 at a time and wins Rs.1 with probability ½. He stops playing if he loses Rs.2 or wins Rs.4. (a)what is the tpm of the related Markov chain? (b) What is the probability that he has lost his money at the end of 5 plays? (c) What is the probability that the same game lasts more than 7 plays?(Pg-456)

10. There are 2 white marbles in urn A and 3 red marbles in urn B. At each step of the process, a marble is selected from each urn and the 2 marbles selected are interchanged. Let the state a_I of the system be the number of red marbles in A after i changes. What is the probability that there are 2 red marbles in urn A?(Pg –457)

11. Three boys A,B and C are throwing a ball to each other. A always throws the ball to B and B always throws the ball to C, but C is just as likely to throw the ball to B as to A. Show that the process is Markovian. Find the transition matrix and classify the states. (Pg – 459).

UNIT IV

QUEUEING THEORY

PART A

1. What are the characteristics of a queueing system?
   Solution: (i) The input pattern
   (ii) The service mechanism
   (iii)The queue discipline

2. What do you mean by transient state and steady-state queueing systems?

Solution: If the characteristics of a queueing system are independent of time or equivalently if the behaviour of the system is independent of time, the system is said to be in steady-state. Otherwise it is said to be in transient-state.

3. Write down the Little's formula that hold good for the infinite capacity Poisson queue models.
   Solution: (i) $E(N_s) = \frac{\lambda}{\mu-\lambda} = \lambda E(W_s)$

   (ii) $E(N_q) = \frac{\lambda^2}{\mu(\mu-\lambda)} = \lambda E(W_q)$

   (iii) $E(W_s) = E(W_q) + \frac{1}{\mu}$

   (iv) $E(N_s) = E(N_q) + \frac{\lambda}{\mu}$

4. If a customer has to wait in a (M/M/1):(∞/FIFO) queue system, What is his average waiting time in the queue, if $\lambda=8$ per hour and $\mu=12$ per hour.
   Solution: 5 min.

5. If there are 2 servers in an infinite capacity Poisson queue system with $\lambda=10$ per hour and $\mu=15$ per hour,what is the percentage of idle time for each server.
   Solution: 50%

6. In a 3 server infinite capacity Poisson queue model if $\lambda/s\mu=2/3$ Find P_0.
   Solution : 1/9
7. If $\lambda/s\mu=2/3$ in a (M/M/s):(∞/FIFO) queue system find the average number of customers in the nonempty queue.
   Solution: 2

8. What is the probability that an arrival to infinite capacity 3 server infinite capacity Poisson queue with $\lambda/s\mu=2/3$ and $P_0=1/9$ enters the service without waiting?
   Solution : 5/9

9. What is the average waiting time of a customer in the 3 server infinite capacity Poisson queue if he happens to wait, given that
λ=6 per hour and μ=4 per hour.
Solution: 10 min

10. If λ=4 per hour and μ=12 per hour in an (M/M/1):(4/FIFO) queueing system, find the probability that there is no customer in the system. If $\lambda=\mu$, what is the value of this probability?
Solution: 81/121 ;1/5.

PART B

Reference : "Probability,Statistics and Queueing Theory"
- T. Veerarajan

1. The local one-person barber shop can accommodate a maximum of 5 people at a time(4 waiting and 1 getting hair cut). Customers arrive according to a Poisson distribution with mean 5 per hour. The barber cuts hair at an average rate of 4 per hour.(Exponential service time).
   (a) What percentage of time is the barber idle?
   (b) What fraction of the potential customers are turned away?
   (c) What is the expected number of customers waiting for a hair cut?
   (d) How much time can a customer expect to spend in the barber shop?(Page no: 506)

2. A bank has two tellers working on savings accounts. The first teller handles withdrawals only. The second teller handles deposits only. It has been found that the service time of 3 min per customer. Depositors are found to arrive in a Poisson fashion throughout the day with mean arrival rate of 16 per hour. Withdrawers also arrive in a Poisson fashion with mean arrival rate of 14 per hour. What would be the effect on the average waiting time for the customers if each teller could handle both withdrawals and deposits. What would be the effect, if this could only be accomplished by increasing the service time to 3.5 min.? (Page no: 500)

3. Customers arrive at a one-man barber shop according to a Poisson process mean interarrival time of 12 min. Customers spend an average of 10 min in the barber's chair.
   (a) What is the expected number of customers in the barber shop and in the queue?
   (b) Calculate the percentage of time an arrival can walk straight into the barber's chair without having to wait.
   © How much time can a customer expect to spend in the barber's shop?
   (e) Management will provide another chair and hire another barber, when a customer's waiting time in the shop exceeds 1.25h. How much must the average rate of arrivals increase to warrant a second barber?
   (f) What is the probability that the waiting time in the system is greater than 30 min?
   (page no: 490)

4. A 2-person barber shop has 5 chairs to accommodate waiting customers. Potential customers, who arrive when all 5 chairs are full, leave without entering barber shop. Customers arrive at the average rate of 4 per hour and spend an average of 12 min in the barber's chair. Compute P_0,P_1,$P_7$$E(N_q)$ and E(W).(page no:510)

5. Derive the difference equations for a Poisson queue system in the steady state.(page no:470)
6. There are 3 typists in an office. Each typist can type an average of 6 letters per hour. If letters arrive for being typed at the rate of 15 letters per hour.
   - What fraction of the time all the typists will be busy?
   - What is the average number of letters waiting to be typed?
   - What is the average time a letter has to spend for waiting and for being typed?
   - What is the probability that a letter will take longer than 20 min. waiting to be typed and being typed?

   Solution: Refer : Probability and Queueing Theory by G. Balaji pg.5.33.

7. Determine the steady state probabilities for M/M/C queueing system.

Solution: Refer : Probability and Queueing Theory by G. Balaji pg.5.52.

8. An automatic car wash facility operates with only one bay. Cars arrive according to a poisson distribution with a mean of 4 cars per hour and may wait in the facility's parking lot if the bay is busy. If the service time for all the cars is constant and equal to 10 minutes, determine
   - Mean number of customers in the system
   - Mean waiting time of a customer in the system
   - Mean waiting time of a customer in the queue
   - Mean number of customers in the queue

   Solution: Refer : Probability and Queueing Theory by G. Balaji pg.5.54.

UNIT V

NON MARKOVIAN QUEUES AND QUEUE NETWORKS

PART A

1. What is the probability then an arrival to an infinite capacity 3 server poisson queuing system with $\frac{\lambda}{\mu} = 2$ and P_o= 1/9 enters the service without waiting.

Solution: P(without waiting) = P(N<3)= $P_o+P_1+P_2$

$$P_n = \frac{1}{n!}\left(\frac{\lambda}{\mu}\right)^n P_o \quad when \quad n \le c = 3$$

$$P(N < 3) = \frac{1}{9} + \frac{2}{9} + \frac{1}{2} \times 2^2 \times \frac{1}{9} = \frac{5}{9}$$

2. Define Little's formula.
` Solution: (i) $E(N_s) = \frac{\lambda}{\mu - \lambda} = \lambda E(W_s)$

(ii) $E(N_q) = \frac{\lambda^2}{\mu(\mu - \lambda)} = \lambda E(W_q)$

(iii) $E(W_s) = E(W_q) + \frac{1}{\mu}$

(iv) $E(N_s) = E(N_q) + \frac{\lambda}{\mu}$

3. Write down the Pollaczek – khinchine formula.
`Solution: The average number of customers in the system is
$$\frac{\lambda^2\sigma^2 + \rho^2}{2(1-\rho)} + \rho$$

4. Define Pollaczek khinchine formula for average queue length.
Solution: Average queue length= $\frac{\lambda^2\sigma^2 + \rho^2}{2(1-\rho)}$

5. Define P-K formula for average waiting time of a customer in the queue.
Solution: Average waiting time of a customer in the queue =
$$\frac{\lambda^2\sigma^2 + \rho^2}{2\lambda(1-\rho)}$$

6. Define P-K formula for average waiting time that a customer spends in the system.
Solution: Average waiting time of a customer in the system =
$$\frac{\lambda^2\sigma^2 + \rho^2}{2\lambda(1-\rho)} + \frac{1}{\mu}$$

PART B

1. Derive the Balance equation of the birth and death process.
Solution: Refer : Probability and Queueing Theory by G. Balaji pg.4.88
2. Derive the Pollaczek- Khinchine formula.

Solution: Refer : Probability and Queueing Theory by G. Balaji pg.5.55.

3. Consider a single server, poisson input queue with mean arrival rate of 10 hour currently the server works according to an exponential distribution with mean service time of 5 minutes. Management has a training course which will result in an improvement in the variance of the service time but at a slight increase in the mean. After completion of the course;, it is estimated that the mean service time will increase to 5.5 minutes but the standard deviation will decrease from 5 minutes to 4 minutes. Management would like to know ;whether they should have the server undergo further training.

   Solution: Refer : Probability and Queueing Theory by G. Balaji pg.5.59.

4. In a heavy machine shop, the overhead crane is 75% utilized. Time study observations gave the average slinging time as 10.5 munutes with a standard deviation of 8.8 minutes. What is the average calling rate for the services of the crane and what is the average delay in getting service? If the average service time is cut to 8.0 minutes, with a standard deviation of 6.0 minutes, how much reduction will occur, on average, in the delay of getting served?

   Solution: Refer : Probability and Queueing Theory by G. Balaji pg.5.61.

5. Automatic car wash facility operates with only on Bay. Cars arrive according to a Poisson process, with mean of 4 cars per hour and may wait in the facility's parking lot if the bay is busy. If the service time for all cars is con;stant and equal to 10 min, determine L_s, L_q, W_s and W_q

   Solution: Refer : Probability and Queueing Theory by G. Balaji pg.5.64.