Chapter 1 Working scientifically

1.1 Questioning and predicting

1.1 Review

1 **a** A hypothesis is a statement that can be tested. This involves making a prediction based on previous observations.

b A theory is a hypothesis that is supported by a great deal of evidence from a wide variety of sources. A principle is a theory that is so strongly supported by evidence that it is unlikely to be shown to be untrue in the future.

2 B. 'What features suggest that sound is a mechanical wave?' is an inquiry question.

3 **a** If the voltage is measured in units of number of batteries, then it is a discrete value.

b If the voltage is measured with a voltmeter, then the voltage would be continuous.

4 qualitative (ordinal)

5 C. A hypothesis should test only one independent variable and it should predict the relationship between the independent and dependent variable. Hypothesis 1 tests two independent variables. Hypothesis 2 does not predict the type of relationship between the independent and dependent variables.

1.2 Planning investigations

1.2 Review

1 B. Repeating experiments and presenting results support the reliability of an experiment.

2 The independent variable is the variable that is changed. In this experiment it is the type of grip (i.e. either two-handed or one-handed).

3 **a** In a controlled experiment, two groups of subjects are tested. The groups, or the tests performed on them, are identical except for a single factor (the variable).

b The dependent variable is the variable that is measured to determine the effect of changes in the independent variable. The independent (experimental) variable is the variable that is changed. For example, in an experiment testing the effect of soil pH on flower colour, the independent variable would be soil pH and the dependent variable would be flower colour.

4 **a** A stopwatch will often be able to measure time to the nearest tenth or hundredth of a second, which could give very precise results. But the accuracy depends on your reaction time, so errors could be introduced.

b A clock can be used to measure time to the nearest seconds, which is less accurate than the stop watch. Again, using this instrument relies on your reaction time, which could introduce errors into your results.

c Using a camera to record the motion means you can pause and slow down the swimmer in order to calculate a more accurate value for the time taken.

5 If you are using a ladder or standing at a great height to drop the balls, this may be a risk.

6 **a** valid

b reliable

c accurate

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1.3 Conducting investigations

1.3 Review

1 Data set A: random error, because there is one unexpected mistake (1.5) in the data.

Data set B: systematic error, because the error is not obvious and may be due to a consistent equipment or operator error.

2 **a** systematic error

b random error

3 If distance = 250 m, time = 16.67 s:

$$\text{speed} = \frac{\text{distance}}{\text{time}}$$
$$= \frac{250}{16.67}\,\frac{\text{m}}{\text{s}}$$
$$= 14.997$$
$$= 15\,\text{m s}^{-1} \text{ (to 2 significant figures)}$$

4 Give your answer to two significant figures, as this is the least number of significant figures in the data provided. Remember to not round off until you have reached the final answer.

5 There could be many reasons why the same experimental results cannot be obtained. The experimental design may be poor because of:

- a lack of objectivity
- a lack of clear and simple instructions
- a lack of appropriate equipment
- a failure to control variables.

Other problems not specifically related to the experiment could be:

- a poor hypothesis that could not be tested objectively
- conclusions that do not agree with the results
- interpretations that are subjective.

6 **a** Systematic error, because this will affect all measurements.

b Random error, because this will affect only some measurements.

1.4 Processing data and information

Worked example: Try yourself 1.4.1

CALCULATING UNCERTAINTY

James is practising his tennis serve. The speed of the ball, in m s^{-1}, is measured to be: 45, 52, 51, 49, 49, 53, 47 Find the mean and uncertainty for these values.

Thinking	Working
Calculate the mean speed.	Mean = (45 + 52 + 51 + 49 + 49 + 53 + 47) ÷ 7 = 49.4 = 49 m s^{-1}
Calculate the maximum variance from the mean.	Maximum difference is 49.4 – 45 = 4.4, so the uncertainty is 4.4.
Write out your average speed and include the uncertainty.	Mean speed is 49 ± 4.4 m s^{-1}.

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1.4 Review

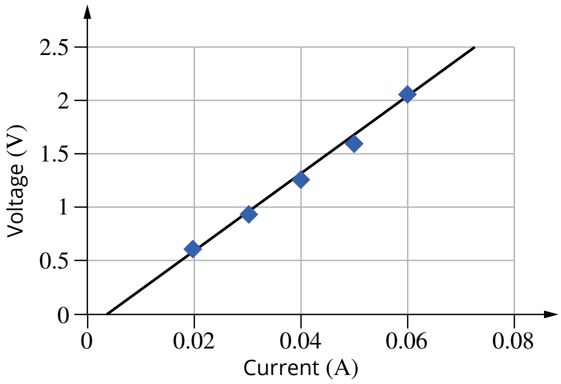
1 An outlier is a data point that does not fit the trend.

2 a mean = (21 + 28 + 19 + 19 + 25 + 24) ÷ 6 = 22.7 = 23

b mode = 19

c median = 22.5

3



4 Add a trend line or line of best fit.

5 The mean is 31 (251 ÷ 8 = 31.375)

The uncertainty is the largest difference between the mean and the data points, which is 37 – 31.4 = 5.6.

6 a 1 ÷ 2 = ±0.5°C

b (0.5 ÷ 20) × 100 = ±2.5%

1.5 Analysing data and information

Worked example: Try yourself 1.5.1

FINDING A LINEAR RELATIONSHIP FROM DATA

The downward force is measured for a variety of different masses.

Mass (kg)	Force (N)
0.25	2.4
0.50	4.9
0.75	7.4
1.0	9.9
1.3	12.8

Find the linear relationship between the values for mass m and force $\vec{F}$.

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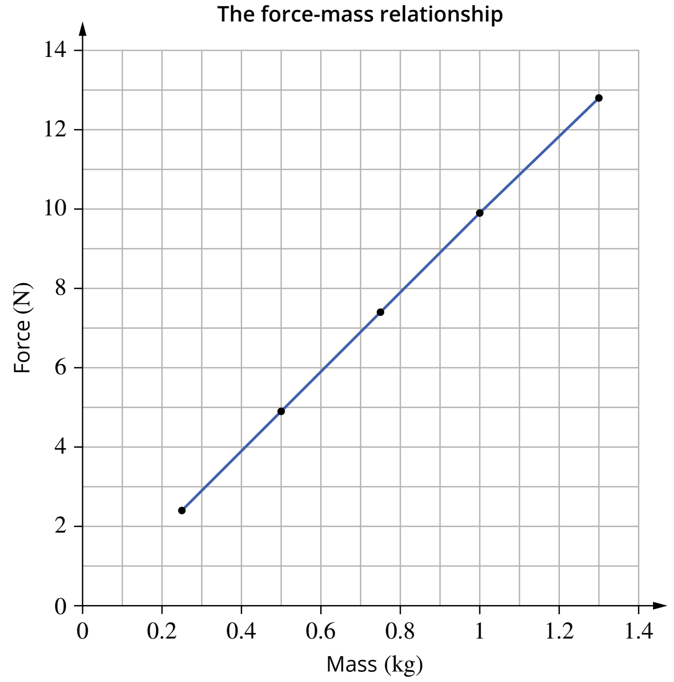
Thinking	Working
Decide which axes each of your variables should be on.	The independent variable is the force, so this will go on the *y*-axis. The dependent variable is the mass, so this will go on the *x*-axis.
Graph your data as a scatter plot and draw a line of best fit.	
Find the line of best fit.	This graph of the data was created on a computer spreadsheet. The line of best fit was created mathematically and plotted. The computer calculated the equation of the line. Graphics calculators can also do this. $y = 9.9202x - 0.0594$
Write out your linear relationship in the form $y = mx + c$.	A scientific calculator or graphics calculator or spreadsheet gives the regression line as $y = 9.9202x - 0.0594$ If this is rearranged and the constants are suitably rounded, the equation is $F = 9.9m - 0.06$ Acceleration is then $9.9\,\mathrm{m\,s^{-2}}$.

Worked example: Try yourself 1.5.2

FINDING A NON-LINEAR RELATIONSHIP FROM DATA

Some students were investigating the relationship between distance and the intensity of sound. They obtained the data shown in the table.

Distance, r (m)	Intensity, I ($\mathrm{W\,m^{-2}}$)
1	0.04
2	0.01
3	0.005
4	0.003
5	0.002

According to the theory they had researched on relevant Internet sites, the students believed that the relationship between I and r is:

$I = P\frac{1}{r^2}$

where P is some constant.

By appropriate manipulation and graphical techniques, find the students' experimental value for P.

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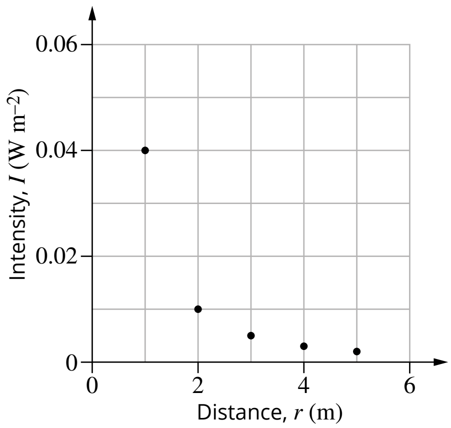
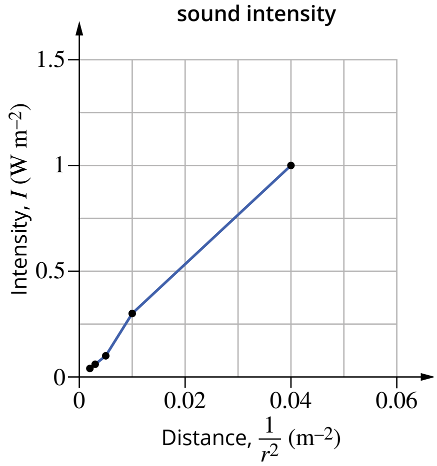
<table>
<tr><th>Thinking</th><th>Working</th></tr>
<tr><td>Plot a graph of the raw data</td><td>
</td></tr>
<tr><td>Work out what you would have to graph to get a straight line.</td><td>You can see what to graph if you think of the equation like this:
$I \;=\; P\frac{1}{r^2} \;+\; 0$
$\uparrow \quad \uparrow\uparrow \quad \uparrow$
$y \;=\; m\,x \;+\; c$
A graph of I on the vertical axis and $\frac{1}{r^2}$ on the horizontal axis would have a gradient equal to P and a vertical axis intercept equal to 0.</td></tr>
<tr><td>Make a new table of the manipulated data.</td><td>The data is manipulated by finding the cube of each of the values for current.
<table>
<tr><th>Distance inverse squared, $\frac{1}{r^2}$ (m^{-2})</th><th>Intensity, I ($W\,m^{-2}$)</th></tr>
<tr><td>1</td><td>0.04</td></tr>
<tr><td>0.3</td><td>0.01</td></tr>
<tr><td>0.1</td><td>0.005</td></tr>
<tr><td>0.06</td><td>0.003</td></tr>
<tr><td>0.04</td><td>0.002</td></tr>
</table></td></tr>
<tr><td>Plot the graph of manipulated data.</td><td>Inverse square relationship of sound intensity

</td></tr>
<tr><td>Calculate the line of best fit.</td><td>This graph of the data was created on a computer spreadsheet. The line of best fit was created mathematically and plotted. The computer calculated the equation of the line. Graphics calculators can also do this.
$y = 0.0395x$</td></tr>
</table>

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Find the equation relating *I* and *r*.	The regression line has the equation $y = 0.04x$, so the equation relating *I* and *r* is $I = 0.04\frac{1}{r^2}$
Write out the value for *P*. Remember to include the correct units.	$P = 0.04$ W

1.5 Review

1. A linear graph shows the proportional relationship between two variables.
2. An inversely proportional relationship.
3. Directly proportional.
4. Time constraints and limited resources.
5. The *y* variable is for the speed of the ball, and the *t* variable is for time. Because this is modelled off $v = u + at$, 9.6 represents the acceleration of the ball in $m\,s^{-2}$ and 1.3 represents its initial speed in $m\,s^{-1}$.

1.6 Problem solving

1.6 Review

1. B. A scientific report should always be written using objective language. A concluding paragraph must summarise the information presented in the report and connect it with the title. It should also include limitations, possible applications of the research and potential future research.
2. From the table, a higher mass coincides with a lower acceleration, which supports the hypothesis.
3. Different objects fell at different speeds. This refutes the hypothesis. You may conclude that:
   - The investigation needs more data in order to find a trend, or your method of finding the results is unreliable.
   - The investigation possibly did not factor in air resistance, and should be repeated with objects with different masses but the same air resistance.
4. The statement ‘Many repeats of the procedure were conducted’ is unquantified. ‘Thirty repeats of the procedure were conducted’ is better because the number of trials is quantified.

1.7 Communicating

1.7 Review

1. B. Scientific writing should not use biased or absolute language.
2. D. First-person narrative uses the pronoun ‘I’.
3. **a** $m\,s^{-2}$
   b N m
   c $kg\,m\,s^{-1}$
   d $kg\,m^{-3}$
4. GW = 10^9 W
   MW = 10^6 W
   $4.5 \times 10^9 = 4.5 \times 10^3 \times 10^6 = 4.5 \times 10^3$ MW
   $= 4500$ MW
5. When using different formulas or comparing results, all the variables will need to be in the same units or else there will be errors in your analysis.

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CHAPTER 1 REVIEW

1 A hypothesis is a testable prediction, based on evidence and prior knowledge, to answer the inquiry question. A hypothesis often takes the form of a proposed relationship between two or more variables.

2 2 Form a hypothesis.
5 Collect results.
3 Plan experiment and equipment.
7 Draw conclusions.
6 Question whether results support hypothesis.
1 State the inquiry question to be investigated.
4 Perform experiment.

3 Elimination, substitution, isolation, engineering controls, administrative controls, personal protective equipment.

4 Dependent variable: flight displacement
Independent variable: release angle
Controlled variable: (any of) release velocity, release height, landing height, air resistance (including wind)

5 **a** the acceleration of the object
b the vertical acceleration of the falling object
c the rate of rotation of the springboard diver

6 $6.8 \pm 0.4\,\text{cm}\,\text{s}^{-1}$

7 the mean

8 a non-linear relationship

9 This graph should show a straight line with a positive gradient.

10 Any issues that could have affected the validity, accuracy, precision or reliability of the data plus any sources of error or uncertainty.

11 Bias is a form of systematic error resulting from a researcher's personal preferences or motivations.

12 C. This option includes the most amount of information.

13 $2500\,\mu\text{m}$

14 Any of the following are correct:
- the activity that you will be carrying out
- where you will be working, e.g. in a laboratory, school grounds, or a natural environment
- how you will use equipment or chemicals
- what clothing you should wear.

15 **a** **i** validity
ii reliability
b The conclusion is not valid because it does not relate to the original hypothesis, which was about liver function, not weight. To be valid, the conclusion must state that there is a relationship (or there is no relationship) between eating fast food and a decrease in liver function.
c To improve the validity of the experiment, the hypothesis should be changed to specify the group that is being studied, i.e. teenage boys. A larger sample size should be used to improve reliability. The group must be divided into an experimental group and control group. The experimental group should only eat fast food and the control group should eat a normal diet. To improve the accuracy of results all measurements should be performed using the same calibrated equipment. Using different equipment can also assist in eliminating systematic errors. Repeat readings of the liver function test should be performed on each sample and the average calculated to minimise the effects of random errors.

16 The purpose of referencing and acknowledgments is to avoid plagiarism and ensure creators and sources are properly credited for their work.

17 Percentage uncertainty $= \left(\frac{0.1}{200}\right) \times 100 = \pm 0.05\%$

18 **a** A controlled variable is a variable which must be kept constant during your investigation. This is so that your results are only affected by changing the independent variable.
b A control experiment is a separate practical investigation that is conducted at the same time where the independent variable is kept constant.

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19

Information	Qualitative	Quantitative
cost $3.95		✓
robust aroma	✓	
coffee temperature 82°C		✓
cup height 9 cm		✓
frothy appearance	✓	
volume 180 mL		✓
strong taste	✓	
white cup	✓	

20 A trend is a pattern or relationship that can be seen between the dependent and independent variables. It may be linear, in which the variables change in direct proportion to each other to produce a straight trend line. The relationship may be in proportion but non-linear, giving a curved trend line. The relationship may also be inverse, in which one variable decreases in response to the other variable increasing. This could be linear or non-linear.

21 Accuracy refers to the ability of the method to obtain the correct measurement close to a true or accepted value. Validity refers to whether an experiment or investigation is actually testing the hypothesis and aims.

22 **a** bar graph
b line graph
c scatter graph (with line of best fit)
d pie chart

23 **a** $1 \div 2 = \pm 0.5°C$
b $(0.5 \div 21) \times 100 = \pm 2.4\%$
$\therefore$ temperature on day 3 = 21°C ± 2.4%
c Mean = $(22 + 24 + 21 + 25 + 27) \div 5 = 23.8 \approx 24$
$\therefore$ uncertainty from the mean = $27 - 23.8 = 3.2 \approx 3$

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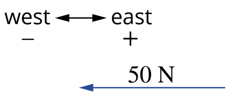
Chapter 2 Motion in a straight line

2.1 Scalars and vectors

Worked example: Try yourself 2.1.1

DESCRIBING VECTORS IN ONE DIMENSION



Describe the vector above using:

a the direction convention shown	
Thinking	**Working**
Identify the magnitude, unit and direction of the vector.	The magnitude is 50 and the unit is N (newtons).
Identify the vector direction according to the direction convention.	The vector is pointing to the west according to the direction convention.
Combine the magnitude, unit and direction.	The vector is 50 N west.

b the sign convention shown.	
Thinking	**Working**
Convert the physical direction to the corresponding mathematical sign.	The direction west is negative.
Combine the mathematical sign with the magnitude and unit.	The vector is –50 N.

Worked example: Try yourself 2.1.2

ADDING VECTORS IN ONE DIMENSION USING ALGEBRA

A box has the following forces acting on it: 16 N up, 22 N down, 4 N up and 17 N down.
Use the sign and direction conventions shown in Figure 2.1.4 to determine the resultant force vector on the box.

Thinking	**Working**
Apply the sign and direction conventions to change the directions to signs.	16 N up = +16 N 22 N down = –22 N 4 N up = +4 N 17 N down = –17 N
Add the magnitudes and their signs together.	Resultant vector = (+16) + (–22) + (+4) + (–17) = –19 N
Refer to the sign and direction convention to determine the direction of the resultant vector.	Negative is down.
State the resultant vector.	The resultant vector is 19 N down.

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Worked example: Try yourself 2.1.3

SUBTRACTING VECTORS IN ONE DIMENSION USING ALGEBRA

A rocket accelerates from 212 m s^{-1} up to 2200 m s^{-1} up. Use the sign and direction conventions shown in Figure 2.1.9 to determine the change in velocity of the rocket.

Thinking	Working
Apply the sign and direction conventions to change the directions to signs.	$\vec{v}_1$ = 212 m s^{-1} up = +212 m s^{-1} $\vec{v}_2$ = 2200 m s^{-1} up = +2200 m s^{-1}
Reverse the direction of the initial velocity $\vec{v}_1$ by reversing the sign.	$-\vec{v}_1$ = 212 m s^{-1} down = –212 m s^{-1}
Use the formula for change in velocity to calculate the magnitude and the sign of $\Delta\vec{v}$.	$\Delta\vec{v} = \vec{v}_2 + (-\vec{v}_1)$ = +2200 + (–212) = +1988 m s^{-1}
Refer to the sign and direction conventions to determine the direction of the change in velocity.	Positive is up.
State the resultant vector.	The change in velocity is 1988 m s^{-1} up.

2.1 Review

1 Using sign conventions, resultant = –2 + 5 – 7 = –4. The resultant vector is 4 m west.

2 Using sign conventions, resultant = +3 – 2 – 3 = –2. The resultant vector is 2 m down.

3 Using sign conventions, resultant = +23 + (–16) + 7 + (–3) = +11. The resultant vector is 11 m forwards.

4 D. Adding vector B to vector A is equivalent to saying A + B. Therefore, draw vector A first, then draw vector B with its tail at the head of A. The resultant vector is drawn from the tail of the first vector (A) to the head of the last vector (B).

5 Change in velocity = final velocity – initial velocity
= 5 – (–3)
= 8 m s^{-1} east

6 Change in velocity = final velocity – initial velocity
= 2 – (+4)
= 2 m s^{-1} left

7 Change in velocity = final velocity – initial velocity
= 3 – (+4)
= 7 m s^{-1} downwards

8 Change in velocity = final velocity – initial velocity
= 8.2 – (+22.2)
= 14.0 m s^{-1} backwards

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2.2 Displacement, speed and velocity

Worked example: Try yourself 2.2.1

AVERAGE VELOCITY AND CONVERTING UNITS

Sally is an athlete performing a training routine by running backwards and forwards along a straight stretch of running track. She jogs 100 m west in a time of 20 s, then turns and walks 160 m east in a further 45 s before stopping.

a What is Sally's average velocity in $m\,s^{-1}$?	
Thinking	**Working**
Calculate the displacement. (Remember that total displacement is the sum of individual displacements. The direction is required.) Sally's total journey consists of two displacements: 100 m west and 160 m east. Take east to be the positive direction.	$\vec{s}$ = sum of displacements = 100 m west + 160 m east = –100 + 160 = + 60 m or 60 m east
Work out the total time taken for the journey.	Time taken = 20 + 45 = 65 s
Substitute the values into the velocity equation.	average velocity $\vec{v}_{av} = \frac{\vec{s}}{\Delta t}$ $= \frac{60}{65}$ $= 0.92\,m\,s^{-1}$
Velocity is a vector, so a direction must be given.	$0.92\,m\,s^{-1}$ east

b What is the magnitude of Sally's average velocity in $km\,h^{-1}$?	
Thinking	**Working**
Convert from $m\,s^{-1}$ to $km\,h^{-1}$ by multiplying by 3.6.	$0.92 \times 3.6 = 3.3\,km\,h^{-1}$ east
As the magnitude of velocity is needed, direction is not required in this answer.	$v_{av} = 3.3\,km\,h^{-1}$

c What is Sally's average speed in $m\,s^{-1}$?	
Thinking	**Working**
Calculate the distance. (Remember that distance is the length of the path covered in the entire journey. The direction does not matter.) Sally travels 100 m in one direction and then 160 m the other way.	$d = 100 + 160$ $= 260$ m
Work out the total time taken for the journey.	20 + 45 = 65 s
Substitute the values into the speed equation.	Distance is 260 m. Time taken is 65 s. average speed $v_{av} = \frac{d}{\Delta t}$ $= \frac{260}{65}$ $= 4.0\,m\,s^{-1}$

d What is Sally's average speed in $km\,h^{-1}$? Give your answer to two significant figures.	
Thinking	**Working**
Convert from $m\,s^{-1}$ to $km\,h^{-1}$ by multiplying by 3.6.	average speed $v_{av} = 4.0\,m\,s^{-1}$ $= 4.0 \times 3.6$ $= 14\,km\,h^{-1}$

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2.2 Review

1 B and C. The distance travelled is $25 \times 10 = 250$ m, but the displacement is zero because the swimmer starts and ends at the same place.

2 **a** Displacement = final position – initial position
$= 40 - 0$
$= +40$ cm
Distance travelled = 40 cm

b Displacement = final position – initial position
$= 40 - 50$
$= -10$ cm
Distance travelled = 10 cm

c Displacement = final position – initial position
$= 70 - 50$
$= +20$ cm
Distance travelled = 20 cm

d Displacement = final position – initial position
$= 70 - 50$
$= +20$ cm
Distance travelled = 50 + 30
= 80 cm

3 **a** $d = 50 + 30 = 80$ km

b $\vec{s}$ = 50 km north + 30 km south
$= 50 + (-30)$
$= 50 - 30$
$= +20$ km or 20 km north

4 **a** The basement is 10 m downwards from the starting position on the ground floor. This can be calculated using the following equation:
$\vec{s}$ = final position – initial position
$= -10 - 0$
$= -10$ m or 10 m downwards

b The total displacement from the basement to the top floor is 60 m upwards. This can be calculated using the following equation:
$\vec{s}$ = final position – initial position
$= +50 - (-10)$
$= 50 + 10$
$= +60$ m or 60 m upwards

c The total distance travelled is 70 m.
$10 + 10 + 50 = 70$ m

d The top floor is 50 m upwards from the starting position on the ground floor. This can be calculated using the following equation:
$\vec{s}$ = final position – initial position
$= 50 - 0$
$= +50$ m or 50 m upwards

5 **a** average speed $v_{av} = \frac{\text{distance travelled}}{\text{time taken}}$
$= \frac{400}{12}$
$= 33\ \text{m s}^{-1}$

b The car travelled 25 m. This can be calculated using the following method:
average speed $v_{av} = \frac{\text{distance travelled}}{\text{time taken}}$
$d = v_{av} \times t$
$= 33 \times 0.75$
$= 25$ m

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6 **a** $90\,\text{min} = \frac{90}{60}$

$= 1.5\,\text{h}$

average speed $v_{av} = \frac{\text{distance travelled}}{\text{time taken}}$

$= \frac{25}{1.5}$

$= 17\,\text{km}\,\text{h}^{-1}$

b To convert from $\text{km}\,\text{h}^{-1}$ to $\text{m}\,\text{s}^{-1}$, you need to divide by 3.6. So:

average speed $v_{av} = \frac{17}{3.6}$

$= 4.7\,\text{m}\,\text{s}^{-1}$

7 **a** average speed $v_{av} = \frac{\text{distance travelled}}{\text{time taken}}$

$= \frac{d}{\Delta t}$

$= \frac{9}{10}$

$= 0.9\,\text{m}\,\text{s}^{-1}$

b Displacement is 1 m east of the starting position.

average velocity $\vec{v}_{av} = \frac{\text{displacement}}{\text{time taken}}$

$= \frac{\vec{s}}{\Delta t}$

$= \frac{1}{10}$

$= 0.1\,\text{m}\,\text{s}^{-1}$ east

8 **a** Distance travelled = 10 km north + 3 km south + x km north to finish 15 km north of the start.

$x = 8$ km north.

Total distance covered = 10 + 3 + 8

= 21 km

b She finishes 15 km north of her starting point. This is her displacement.

c average speed $v_{av} = \frac{\text{distance travelled}}{\text{time taken}}$

$= \frac{21}{1.5}$

$= 14\,\text{km}\,\text{h}^{-1}$

d average velocity $\vec{v}_{av} = \frac{\text{displacement}}{\text{time taken}}$

$= \frac{15}{1.5}$

$= 10\,\text{km}\,\text{h}^{-1}$ north

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2.3 Acceleration

Worked example: Try yourself 2.3.1

CHANGE IN SPEED AND VELOCITY PART 1

A ball is dropped onto a concrete floor and strikes the floor at $9.0\,m\,s^{-1}$. It then rebounds at $7.0\,m\,s^{-1}$.

a What is the change in speed of the ball?	
Thinking	**Working**
Find the values for the initial speed and the final speed of the ball.	$u = 9.0\,m\,s^{-1}$ $v = 7.0\,m\,s^{-1}$
Substitute the values into the change in speed equation: $\Delta v = v - u$	$\Delta v = v - u$ $= 7.0 - 9.0$ $= -2.0\,m\,s^{-1}$ (Speed is a scalar, so the negative value indicates a decrease in magnitude, as opposed to a negative direction.)

b What is the change in velocity of the ball?	
Thinking	**Working**
Velocity is a vector. Apply the sign convention to replace the directions.	$\vec{u} = 9.0\,m\,s^{-1}$ downwards $= -9.0\,m\,s^{-1}$ $\vec{v} = 7.0\,m\,s^{-1}$ upwards $= +7.0\,m\,s^{-1}$
As this is a vector subtraction, reverse the direction of $\vec{u}$ to get $-\vec{u}$.	$\vec{u} = -9.0\,m\,s^{-1}$ $-\vec{u} = 9.0\,m\,s^{-1}$
Substitute the values into the change in velocity equation: $\Delta\vec{v} = \vec{v} + (-\vec{u})$	$\Delta\vec{v} = \vec{v} + (-\vec{u})$ $= 7.0 + (+9.0)$ $= 16.0\,m\,s^{-1}$
Apply the sign convention to describe the direction.	$\Delta\vec{v} = 16\,m\,s^{-1}$ upwards

Worked example: Try yourself 2.3.2

CHANGE IN SPEED AND VELOCITY PART 2

A ball is dropped onto a concrete floor and strikes the floor at $9.0\,m\,s^{-1}$. It then rebounds at $7.0\,m\,s^{-1}$. The contact time with the floor is 35 ms. What is the average acceleration of the ball during its contact with the floor?	
Thinking	**Working**
Note the values you will need in order to find the average acceleration (initial velocity, final velocity and time). Convert ms into s by dividing by 1000. (Note that $\Delta\vec{v}$ was calculated for this situation in the previous Worked example.)	$\vec{u} = -9.0\,m\,s^{-1}$ $-\vec{u} = 9.0\,m\,s^{-1}$ $\vec{v} = 7.0\,m\,s^{-1}$ $\Delta\vec{v} = 16\,m\,s^{-1}$ upwards $\Delta t = 35\,ms$ $= 0.035\,s$
Substitute the values into the average acceleration equation.	$\vec{a}_{av} = \frac{\text{change in velocity}}{\text{time taken}}$ $= \frac{\Delta\vec{v}}{\Delta t}$ $= \frac{16}{0.035}$ $= 457 = 460\,m\,s^{-2}$
Acceleration is a vector, so you must include a direction in your answer.	$\vec{a}_{av} = 460\,m\,s^{-2}$ upwards (to 2 significant figures)

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2.3 Review

1 $\Delta v = v - u$

$= 3 - 10$

$= -7\,\text{km}\,\text{h}^{-1}$

Note that speed is a scalar, so the negative value indicates a decrease in magnitude, not a negative direction.

2 $\Delta\vec{v} = \vec{v} - \vec{u}$

$= 0 - (-5)$

$= +5\,\text{m}\,\text{s}^{-1}$ or $5\,\text{m}\,\text{s}^{-1}$ upwards

3 Down is negative, so the initial velocity is $-5.0\,\text{m}\,\text{s}^{-1}$.

$\Delta\vec{v} = \vec{v} - \vec{u}$

$= +3.0 - (-5.0)$

$= +8\,\text{m}\,\text{s}^{-1}$

$= 8\,\text{m}\,\text{s}^{-1}$ upwards

4 $\vec{a}_{av} = \frac{\text{change in velocity}}{\text{time taken}}$

$= \frac{\Delta\vec{v}}{\Delta t}$

$= \frac{0-7.5}{1.5}$

$= -5.0\,\text{m}\,\text{s}^{-2}$

$= 5.0\,\text{m}\,\text{s}^{-2}$ south

5 $\vec{a}_{av} = \frac{\text{change in velocity}}{\text{time taken}}$

$= \frac{\Delta\vec{v}}{\Delta t}$

$= \frac{150-0}{3.5}$

$= 43\,\text{km}\,\text{h}^{-1}\,\text{s}^{-1}$

6 **a** $\Delta v = v - u$ (only magnitudes required)

$= 15 - 25$

$= -10\,\text{m}\,\text{s}^{-1}$

Note that speed is a scalar, so the negative value indicates a decrease in magnitude, not a negative direction.

b $\Delta\vec{v} = \vec{v} - \vec{u}$

$= -15 - (+25)$

$= -40\,\text{m}\,\text{s}^{-1}$

$= 40\,\text{m}\,\text{s}^{-1}$ west

c $\vec{a}_{av} = \frac{\text{change in velocity}}{\text{time taken}}$

$= \frac{\Delta\vec{v}}{\Delta t}$

$= \frac{40}{0.050}$

$= 800\,\text{m}\,\text{s}^{-2}$

Note that only the magnitude was required so no direction is given.

7 **a** $\Delta v = v - u$ (only magnitudes required)

$= 8.0 - 0$

$= 8.0\,\text{m}\,\text{s}^{-1}$

b $\Delta\vec{v} = \vec{v} - \vec{u}$

$= -8.0 - 0$

$= -8.0\,\text{m}\,\text{s}^{-1}$

$= 8.0\,\text{m}\,\text{s}^{-1}$ south

c $\vec{a}_{av} = \frac{\text{change in velocity}}{\text{time taken}}$

$= \frac{\Delta\vec{v}}{\Delta t}$

$= \frac{8.0}{1.2}$

$= 6.7\,\text{m}\,\text{s}^{-2}$, south

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2.4 Graphing position, velocity and acceleration over time

Worked example: Try yourself 2.4.1

ANALYSING A POSITION–TIME GRAPH

Use the graph shown in Worked example 2.4.1 to answer the following questions.

a What is the velocity of the cyclist between E and F?

Thinking	Working
Determine the change in position (displacement) of the cyclist between E and F using: $\vec{s}$ = final position – initial position	At E, $x = 300\,\text{m}$. At F, $x = 0\,\text{m}$. $\vec{s} = 0 - 300$ $= -300\,\text{m}$ or $300\,\text{m}$ backwards (That is, back towards the starting point.)
Determine the time taken to travel from E to F.	$t = 100 - 80$ $= 20\,\text{s}$
Calculate the gradient of the graph between E and F using: gradient of x–t graph $= \frac{\text{rise}}{\text{run}} = \frac{\Delta x}{\Delta t}$ Remember that $\Delta x = \vec{s}$.	Gradient $= \frac{-300}{20}$ $= -15$
State the velocity, using: gradient of x–t graph = velocity Velocity is a vector so direction must be given.	Since the gradient is –15, the velocity is $-15\,\text{m}\,\text{s}^{-1}$ or $15\,\text{m}\,\text{s}^{-1}$ backwards (towards the starting point).

b Describe the motion of the cyclist between D and E.

Thinking	Working
Interpret the shape of the graph between D and E.	The graph is flat between D and E, indicating that the cyclist's position is not changing for this time. So the cyclist is not moving. If the cyclist is not moving, the velocity is $0\,\text{m}\,\text{s}^{-1}$.
You may confirm the result by calculating the gradient of the graph between D and E using: gradient of x–t graph $= \frac{\text{rise}}{\text{run}} = \frac{\Delta x}{\Delta t}$ Remember that $\Delta x = \vec{s}$.	Gradient $= \frac{0}{20}$ $= 0$

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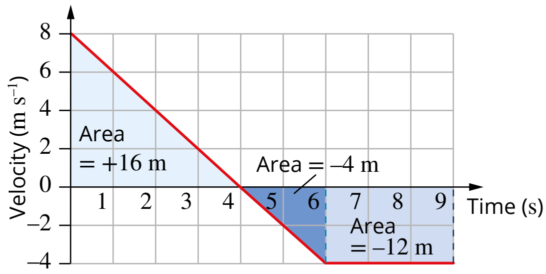
Worked example: Try yourself 2.4.2

ANALYSING A VELOCITY–TIME GRAPH

Use the graph shown in Worked example 2.4.2 to answer the following questions.

a What is the displacement of the car from 4 to 6 seconds?	
Thinking	**Working**
Displacement is the area under the graph. So, calculate the area under the graph for the time period for which you want to find the displacement. Use displacement = $b \times h$ for squares and rectangles. Use displacement = $\frac{1}{2}\, b \times h$ for triangles.	 Area is triangular: area = $\frac{1}{2} \times 2 \times -4$ $= -4$ m
Displacement is a vector quantity, so a direction is needed.	displacement = 4 m west

b What is the average velocity of the car from 4 to 6 seconds?	
Thinking	**Working**
Identify the equation and variables, and apply the sign convention.	$\vec{v} = \frac{\vec{s}}{\Delta t}$ $\vec{s} = -4$ m $\Delta t = 2$ s
Substitute values into the equation: $\vec{v} = \frac{\vec{s}}{\Delta t}$	$\vec{v} = \frac{\vec{s}}{\Delta t}$ $= \frac{-4}{2}$ $= -2\text{ m s}^{-1}$
Velocity is a vector quantity, so a direction is needed.	$\vec{v}_{av} = 2\text{ m s}^{-1}$ west

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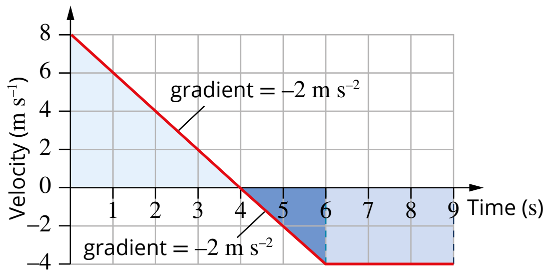
Worked example: Try yourself 2.4.3

FINDING ACCELERATION USING A *V–T* GRAPH

<table>
<tr><td colspan="2">Use the graph shown in Worked example 2.4.3 to answer the following question. What is the acceleration of the car during the period from 4 to 6 seconds?</td></tr>
<tr><td>Thinking</td><td>Working</td></tr>
<tr><td>Acceleration is the gradient of a v–t graph. Calculate the gradient using:
gradient = $\frac{\text{rise}}{\text{run}}$</td><td>

Gradient from 4 to 6 = $\frac{\text{rise}}{\text{run}}$
$= \frac{-4}{2}$
$= -2\,\text{m}\,\text{s}^{-2}$</td></tr>
<tr><td>Acceleration is a vector quantity, so a direction is needed.
Note: In this case, the car is moving in the negative easterly direction and speeding up.</td><td>The acceleration is $2\,\text{m}\,\text{s}^{-2}$ west.</td></tr>
</table>

2.4 Review

1 D. The gradient is the displacement divided by the time taken, which is velocity.

2 The car initially moves in a positive direction and travels 8 m in 2 s. It then stops for 2 s. The car then reverses direction for 5 s, passing back through its starting point after 8 s. It travels a further 2 m in a negative direction before stopping after 9 s.

3 Reading from graph:

a +8 m

b +8 m

c +4 m

d –2 m

4 The car returns to its starting point when the position is zero again, which occurs at $t = 8$ s.

5 **a** The velocity during the first 2 s is equal to the gradient of the graph during this interval.

velocity $= \frac{\text{rise}}{\text{run}} = \frac{8-0}{2} = +4\,\text{m}\,\text{s}^{-1}$

b At 3 s the velocity is zero, because the gradient of the graph = 0.

c Velocity = gradient of graph $= \frac{\text{rise}}{\text{run}} = \frac{>0-8}{4} = -2\,\text{m}\,\text{s}^{-1}$

d The velocity at 8 s is $= -2\,\text{m}\,\text{s}^{-1}$, because the car is travelling at a constant velocity of $-2\,\text{m}\,\text{s}^{-1}$ between 4 s and 9 s.

e The velocity from 8 s to 9 s $= -2\,\text{m}\,\text{s}^{-1}$, since the car is travelling at a constant velocity of $-2\,\text{m}\,\text{s}^{-1}$ between 4 s and 9 s.

6 **a** Distance = 8 + 8 + 2 = 18 m

b Displacement = (–2) – 0 = –2 m

7 **a** Acceleration = gradient

$= \frac{\text{rise}}{\text{run}}$

$= \frac{8}{4}$

$= 2\,\text{m}\,\text{s}^{-2}$

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b The bus will overtake the bicycle when they have both travelled the same distance, given by the areas under the two graphs. After 8 seconds the bus has travelled 56 metres and the bicycle has travelled 64 metres. After 10 seconds both the bus and the bicycle have travelled 80 metres.

Algebraically, this could be determined by:

The displacement for the bus = 56 + 12 (t – 8)

The displacement for the bike = 8t

Equating these two displacements gives:

$8t = 56 + 12t - 96$

$12t - 8t = 96 - 56$

$4t = 40$

$t = 10\,\text{s}$

c After 10 s the bike has travelled $10 \times 8 = 80\,\text{m}$.

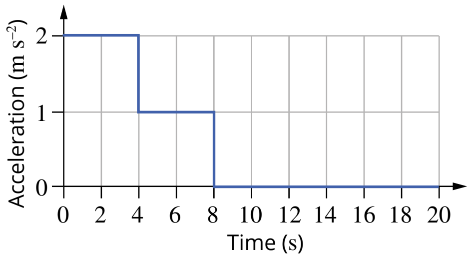
d Average velocity $\vec{v}_{av} = \frac{\text{displacement}}{\text{time taken}}$

$$\vec{s} = \left(\tfrac{1}{2} \times 4 \times 8\right) + (4 \times 8) + \left(\tfrac{1}{2} \times 4 \times 4\right)$$
$$= 16 + 32 + 8$$
$$= 56\,\text{m forwards}$$

So

$$\vec{v}_{av} = \frac{56}{8}$$
$$= 7\,\text{m}\,\text{s}^{-1}\text{ forwards}$$

8 **a**



b The change in velocity of the bus over the first 8 s is determined by calculating the area under the acceleration–time graph from $t = 0$ to $t = 8\,\text{s}$, i.e. $+12\,\text{m}\,\text{s}^{-1}$.

2.5 Equations of motion

Worked example: Try yourself 2.5.1

USING THE EQUATIONS OF MOTION

A snowboarder in a race is travelling $15\,\text{m}\,\text{s}^{-1}$ east as she crosses the finishing line. She then decelerates uniformly until coming to a stop over a distance of 30 m.

a What is her acceleration as she comes to a stop?	
Thinking	**Working**
Write down the known quantities as well as the quantity that you are finding. Apply the sign convention that east is positive and west is negative.	$\vec{s} = +30\,\text{m}$ $\vec{u} = +15\,\text{m}\,\text{s}^{-1}$ $\vec{v} = 0\,\text{m}\,\text{s}^{-1}$ $\vec{a} = ?$
Identify the correct equation to use.	$\vec{v}^2 = \vec{u}^2 + 2\vec{a}\vec{s}$
Substitute known values into the equation and solve for $\vec{a}$. Include units with the answer.	$\vec{v}^2 = \vec{u}^2 + 2\vec{a}\vec{s}$ $0^2 = 15^2 + 2 \times \vec{a} \times 30$ $\vec{a} = \frac{0 - 225}{60}$ $= -3.8\,\text{m}\,\text{s}^{-2}$
Use the sign convention to state the answer with its direction.	$\vec{a} = 3.8\,\text{m}\,\text{s}^{-2}$ west

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b How long does she take to come to a stop?	
Thinking	**Working**
Write down the known quantities as well as the quantity that you are finding. Apply the sign convention that east is positive and west is negative.	$\vec{s} = 30\,\text{m}$ $\vec{u} = 15\,\text{m}\,\text{s}^{-1}$ $\vec{v} = 0\,\text{m}\,\text{s}^{-1}$ $\vec{a} = -3.8\,\text{m}\,\text{s}^{-2}$ $t = ?$
Identify the correct equation to use. Since you now know four values, any equation involving *t* will work.	$\vec{v} = \vec{u} + \vec{a}t$
Substitute known values into the equation and solve for *t*. Include units with the answer.	$t = \frac{0-15}{-3.8}$ $= 4.0\,\text{s}$

c What is the average velocity of the snowboarder as she comes to a stop?	
Thinking	**Working**
Write down the known quantities as well as the quantity that you are finding. Apply the sign convention that east is positive and west is negative.	$\vec{u} = +15\,\text{m}\,\text{s}^{-1}$ $\vec{v} = 0\,\text{m}\,\text{s}^{-1}$ $\vec{v}_{av} = ?$
Identify the correct equation to use.	$\vec{v}_{av} = \frac{1}{2}(\vec{u} + \vec{v})$
Substitute known quantities into the equation and solve for $\vec{v}_{av}$. Include units with the answer.	$\vec{v}_{av} = \frac{1}{2}(\vec{u} + \vec{v})$ $= \frac{1}{2}(15 + 0)$ $= 7.5\,\text{m}\,\text{s}^{-1}$
Use the sign convention to state the answer with its direction.	$\vec{v}_{av} = 7.5\,\text{m}\,\text{s}^{-1}$ east

2.5 Review

1 E. You have been given the initial speed, final speed and distance travelled, and you are asked for the acceleration. So the chosen equation must contain $\vec{s}$, $\vec{u}$, $\vec{v}$ and $\vec{a}$.

2 **a** $\vec{u} = 0\,\text{m}\,\text{s}^{-1}$, $\vec{s} = 400\,\text{m}$, $t = 16\,\text{s}$, $\vec{a} = ?$

$$\vec{s} = \vec{u}t + \tfrac{1}{2}\vec{a}t^2$$

$$400 = 0 + \tfrac{1}{2}\vec{a} \times 162$$

$$\vec{a} = \tfrac{400}{256} \times 2$$

$$= 3.1\,\text{m}\,\text{s}^{-2} \text{ forward}$$

b $\vec{u} = 0\,\text{m}\,\text{s}^{-1}$, $\vec{s} = 400\,\text{m}$, $t = 16\,\text{s}$, $\vec{a} = 3.1\,\text{m}\,\text{s}^{-2}$, $\vec{v} = ?$

$\vec{v} = \vec{u} + \vec{a}t$

$= 0 + 3.1 \times 16$

$v = 50\,\text{m}\,\text{s}^{-1}$

(Direction is not required in this answer.)

c To convert $\text{m}\,\text{s}^{-1}$ to $\text{km}\,\text{h}^{-1}$, multiply by 3.6.

$50\,\text{m}\,\text{s}^{-1} \times 3.6 = 180\,\text{km}\,\text{h}^{-1}$

(Direction is not required in this answer.)

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3 **a** $\vec{u} = 0\,\text{m s}^{-1}$, $t = 8.0\,\text{s}$, $\vec{v} = 16\,\text{m s}^{-1}$, $\vec{a} = ?$

$\vec{v} = \vec{u} + \vec{a}t$

$16 = 0 + a \times 8.0$

$a = \frac{16}{8.0}$

$= +2.0\,\text{m s}^{-2}$

b $\vec{v}_{av} = \frac{\vec{u}+\vec{v}}{2}$

$= \frac{0+16}{2}$

$= +8\,\text{m s}^{-1}$

c $\vec{u} = 0\,\text{m s}^{-1}$, $t = 8.0\,\text{s}$, $\vec{v} = 16\,\text{m s}^{-1}$, $\vec{a} = 2.0\,\text{m s}^{-2}$, $\vec{s} = ?$

$\vec{s} = \frac{1}{2}(\vec{u} + \vec{a}t)$

$= \frac{1}{2}(0 + 16) \times 8.0$

$s = 64\,\text{m}$

(Direction is not required in this answer.)

4 **a** $\vec{u} = 0\,\text{m s}^{-1}$, $\vec{v} = 160\,\text{m s}^{-1}$ upwards, $t = 4.0\,\text{s}$, $\vec{a} = ?$

$\vec{v} = \vec{u} + \vec{a}t$

$160 = 0 + \vec{a} \times 4.0$

$\vec{a} = 40\,\text{m s}^{-2}$ upwards

b In the first 4.0 s: $\vec{u} = 0\,\text{m s}^{-1}$, $t = 4.0\,\text{s}$, $\vec{v} = 160\,\text{m s}^{-1}$, $\vec{a} = 40\,\text{m s}^{-2}$, $\vec{s} = ?$

$\vec{s} = \vec{u}t + \frac{1}{2}\vec{a}t^2$

$= \frac{1}{2}(0 + 160) \times 4.0$

$= 80 \times 4.0$

$= 320\,\text{m}$

In the last 5.0 seconds: $\vec{u} = 160\,\text{m s}^{-1}$ upwards, $t = 5.0\,\text{s}$, $\vec{v} = 160\,\text{m s}^{-1}$ upwards, $\vec{a} = 0\,\text{m s}^{-2}$, $\vec{s} = ?$

$\vec{s} = \frac{1}{2}(\vec{u} + \vec{v})t$

$= \frac{1}{2}(160 + 160) \times 5.0$

$= 160 \times 5.0$

$= 800\,\text{m}$

Total distance in 9.0 s:

$\vec{s} = 320 + 800$

$= 1120\,\text{m}$ or $1.1\,\text{km}$ (no direction required for distance)

c To convert m s^{-1} to km h^{-1}, multiply by 3.6.

$160\,\text{m s}^{-1} \times 3.6 = 576 = 580\,\text{km h}^{-1}$

d $\vec{u} = 0\,\text{m s}^{-1}$, $\vec{v} = 160\,\text{m s}^{-1}$ upwards

$v_{av} = \frac{u+v}{2}$

$= \frac{0+160}{2}$

$= 80\,\text{m s}^{-1}$ (no direction required for speed)

e $v_{av} = \frac{d}{t}$

$= \frac{1120}{9.0}$

$= 124\,\text{m s}^{-1}$ (no direction required for speed)

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5 **a** $\vec{u} = 4.2\,\text{m s}^{-1}$, $t = 0.5\,\text{s}$, $\vec{v} = 6.7\,\text{m s}^{-1}$, $\vec{a} = ?$

$\vec{v} = \vec{u} + \vec{a}t$

$6.7 = 4.2 + a \times 0.50$

$\vec{a} = \frac{6.7-4.2}{0.50}$

$= 5.0\,\text{m s}^{-2}$, east

b $\vec{u} = 4.2\,\text{m s}^{-1}$, $t = 0.5$, $\vec{v} = 6.7\,\text{m s}^{-1}$, $\vec{a} = 5.0\,\text{m s}^{-2}$, $\vec{s} = ?$

$\vec{s} = \frac{1}{2}(\vec{u} + \vec{v})t$

$= \frac{1}{2}(4.2 + 6.7) \times 0.50$

$= 2.73 = 2.7\,\text{m}$ (no direction required for distance)

c $v_{\text{av}} = \frac{u+v}{2}$

$= \frac{4.2+6.7}{2}$

$= 5.45 = 5.5\,\text{m s}^{-1}$ (no direction required for speed)

6 D. The stone is travelling downwards, so the velocity is downwards. When the stone strikes the water it decelerates, so the acceleration is upwards.

7 **a** $\vec{u} = +4.4\,\text{m s}^{-1}$, $\vec{v} = 0\,\text{m s}^{-1}$, $\vec{s} = +4.0\,\text{m}$, $\vec{a} = ?$

$v^2 = u^2 + 2\vec{a}\vec{s}$

$0 = (4.4)^2 + 2 \times \vec{a} \times 4.0$

$\vec{a} = -\frac{19.36}{8}$

$= -2.4\,\text{m s}^{-2}$ downwards

b $\vec{u} = +4.4\,\text{m s}^{-1}$, $\vec{v} = 0\,\text{m s}^{-1}$, $\vec{s} = +4.0\,\text{m}$, $\vec{a} = -2.4\,\text{m s}^{-2}$, $t = ?$

$\vec{v} = \vec{u} + \vec{a}t$

$0 = 4.4 - 2.4t$

$t = \frac{4.4}{2.4}$

$= 1.8\,\text{s}$

c $\vec{u} = +4.4\,\text{m s}^{-1}$, $\vec{s} = +2.0\,\text{m}$, $\vec{a} = -2.4\,\text{m s}^{-2}$, $\vec{v} = ?$

$v^2 = u^2 + 2\vec{a}\vec{s}$

$= (4.4)^2 + 2 \times -2.4 \times 2.0$

$= 19.36 - 9.68$

$= 9.68$

$\vec{v} = \sqrt{9.68}$

$= 3.1\,\text{m s}^{-1}$ downwards

8 **a** To convert km h^{-1} to m s^{-1}, divided by 3.6.

$\frac{75\,\text{kmh}^{-1}}{3.6} = 20.83 = 21\,\text{m s}^{-1}$

b $\vec{u} = 21\,\text{m s}^{-1}$, $\vec{a} = 0\,\text{m s}^{-2}$, $t = 0.25\,\text{s}$, $\vec{s} = ?$

$\vec{s} = \vec{u}t + \frac{1}{2}\vec{a}t^2$

$= 21 \times 0.25$

$= 5.2\,\text{m}$ (no direction required for distance)

c $\vec{u} = 21\,\text{m s}^{-1}$, $\vec{a} = -6.0\,\text{m s}^{-2}$, $\vec{v} = 0\,\text{m s}^{-1}$, $\vec{s} = ?$

$\vec{v}^2 = \vec{u}^2 + 2\vec{a}\vec{s}$

$0 = (21)^2 + 2 \times -6.0 \times \vec{s}$

$\vec{s} = \frac{(21)^2}{12}$

$= 36.2 = 36\,\text{m}$ (no direction required for distance)

d $5.2 + 36.2 = 41.4 = 41\,\text{m}$

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2.6 Vertical motion

Worked example: Try yourself 2.6.1

VERTICAL MOTION

A construction worker accidentally knocks a hammer from a building, and it falls vertically a distance of 60 m to the ground. Use $\vec{g} = -9.8\,\text{m s}^{-2}$ and ignore air resistance when answering these questions.

a How long does the hammer take to fall halfway, to 30 m?

Thinking	Working
Write down the values of the quantities that are known and what you are finding. Apply the sign convention that up is positive and down is negative.	$\vec{s} = -30\,\text{m}$ $\vec{u} = 0\,\text{m s}^{-1}$ $\vec{a} = -9.8\,\text{m s}^{-2}$ $t = ?$
Identify the equation for uniform acceleration that best fits the data you have.	$\vec{s} = \vec{u}t + \frac{1}{2}\vec{a}t^2$
Substitute known values into the equation and solve for *t*. Think about whether the value seems reasonable.	$-30 = 0 \times t + \frac{1}{2} \times -9.8 \times t^2$ $-30 = -4.9t^2$ $t = \sqrt{\frac{-30}{-4.9}}$ $= 2.5\,\text{s}$

b How long does the hammer take to fall all the way to the ground?

Thinking	Working
Write down the values of the quantities that are known and what you are finding. Apply the sign convention that up is positive and down is negative.	$\vec{s} = -60\,\text{m}$ $\vec{u} = 0\,\text{m s}^{-1}$ $\vec{a} = -9.8\,\text{m s}^{-2}$ $t = ?$
Identify the equation for uniform acceleration that best fits the data you have.	$\vec{s} = \vec{u}t + \frac{1}{2}\vec{a}t^2$
Substitute known values into the equation and solve for *t*. Think about whether the value seems reasonable.	$-60 = 0 \times t + \frac{1}{2} \times (-9.8) \times t^2$ $-60 = -4.9t^2$ $t = \sqrt{\frac{-60}{-4.9}}$ $= 3.5\,\text{s}$

c What is the velocity of the hammer as it hits the ground?

Thinking	Working
Write down the values of the quantities that are known and what you are finding. Apply the sign convention that up is positive and down is negative.	$\vec{s} = -60\,\text{m}$ $\vec{u} = 0\,\text{m s}^{-1}$ $\vec{v} = ?$ $\vec{a} = -9.8\,\text{m s}^{-1}$ $t = 3.5\,\text{s}$
Identify the correct equation to use. Since you now know four values, any equation involving $\vec{v}$ will work.	$\vec{v} = \vec{u} + \vec{a}t$
Substitute known values into the equation and solve for $\vec{v}$. Think about whether the value seems reasonable.	$\vec{v} = 0 + (-9.8) \times 3.5$ $= -34\,\text{m s}^{-1}$
Use the sign and direction convention to describe the direction of the final velocity.	$\vec{v} = -34\,\text{m s}^{-1}$ or $34\,\text{m s}^{-1}$ downwards

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Worked example: Try yourself 2.6.2

MAXIMUM HEIGHT PROBLEMS

On winning a cricket match, a fielder throws a cricket ball vertically into the air at 15.0 m s^{-1}. In this example, air resistance can be ignored and the acceleration due to gravity is –9.80 m s^{-2}.

a Determine the maximum height reached by the ball above its starting position.	
Thinking	**Working**
Write down the values of the quantities that are known and what you are finding. At the maximum height the velocity is zero. Apply the sign convention that up is positive and down is negative.	$\vec{u}$ = 15.0 m s^{-1} $\vec{v}$ = 0 $\vec{a}$ = –9.80 m s^{-2} $\vec{s}$ = ?
Select an appropriate formula.	$\vec{v}^2 = \vec{u}^2 + 2\vec{a}\vec{s}$
Substitute known values into the equation and solve for $\vec{s}$.	$0 = (15)^2 + 2 \times (-9.8) \times \vec{s}$ $\vec{s} = \frac{-225}{-19.6}$ $\therefore\ \vec{s}$ = 11.48 m, i.e. the ball reaches a height of 11 m above its starting position.

b Calculate the time that the ball takes to return to its starting position.	
Thinking	**Working**
To work out the time for which the ball is in the air, it is often necessary to first calculate the time that it takes to reach its maximum height. Write down the values of the quantities that are known and what you are finding.	$\vec{u}$ = 15.0 m s^{-1} $\vec{v}$ = 0 m s^{-1} $\vec{a}$ = –9.80 m s^{-2} $\vec{s}$ = 11.5 m t = ?
Select an appropriate formula.	$\vec{v} = \vec{u} + \vec{a}t$
Substitute known values into the equation and solve for t.	$0 = 15 + (-9.8 \times t)$ $9.8t = 15$ $\therefore\ t$ = 1.53 s The ball takes 1.53 s to reach its maximum height. It will therefore take 1.53 s to fall from this height back to its starting position, so the whole journey will last for 3.06 s.

2.6 Review

1 B. The acceleration of a falling object is due to gravity, so it is constant.

2 A and D. Acceleration due to gravity is constant (down), but velocity changes throughout the journey and is zero at the top of his flight.

3 **a** $\vec{u}$ = 0 m s^{-1}, $\vec{a}$ = –9.8 m s^{-2}, t = 3 s, $\vec{v}$ = ?

$\vec{v} = \vec{u} + \vec{a}t$

$= 0 + (-9.8) \times 3.0$

= 29 m s^{-1} (no direction required for speed)

b $\vec{s}$ = –30 m, $\vec{u}$ = 0 m s^{-1}, $\vec{a}$ = –9.8 m s^{-2}, $\vec{v}$ = ?

$\vec{v}^2 = \vec{u}^2 + 2\vec{a}\vec{s}$

$= 0 + 2 \times (-9.8) \times (-30)$

$v = \sqrt{588}$

= 24 m s^{-1} (no direction required for speed)

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c $\vec{v}_{av} = \frac{1}{2}(\vec{u} + \vec{v})$

$= \frac{1}{2}(0 + 24)$

$= 12\,\text{m}\,\text{s}^{-1}$ downwards

4 **a** $\vec{v} = 0\,\text{m}\,\text{s}^{-1}$, $\vec{a} = -9.8\,\text{m}\,\text{s}^{-2}$, $t = 1.5\,\text{s}$, $\vec{u} = ?$

$\vec{v} = \vec{u} + \vec{a}t$

$0 = \vec{u} - 9.8 \times 1.5$

$\vec{u} = 14.7$ or $15\,\text{m}\,\text{s}^{-1}$ upwards

b $\vec{u} = 14.7\,\text{m}\,\text{s}^{-1}$, $\vec{v} = 0\,\text{m}\,\text{s}^{-1}$, $\vec{a} = -9.8\,\text{m}\,\text{s}^{-2}$, $t = 1.5\,\text{s}$, $\vec{s} = ?$

$\vec{s} = \frac{1}{2}(\vec{u} + \vec{v})t$

$= \frac{1}{2}(14.7 + 0) \times 1.5$

$= +11.0\,\text{m}$

5 **a** $\vec{u} = 0\,\text{m}\,\text{s}^{-1}$, $\vec{a} = -9.8\,\text{m}\,\text{s}^{-2}$, $t = 0.40\,\text{s}$, $\vec{v} = ?$

$\vec{v} = \vec{u} + \vec{a}t$

$= 0 - 9.8 \times 0.40$

$= 3.9\,\text{m}\,\text{s}^{-1}$ (no direction required for speed)

b $\vec{u} = 0\,\text{m}\,\text{s}^{-1}$, $\vec{a} = -9.8\,\text{m}\,\text{s}^{-2}$, $t = 0.40\,\text{s}$, $\vec{v} = -3.9\,\text{m}\,\text{s}^{-1}$, $\vec{s} = ?$

The height from which the book fell is the opposite of the distance it travelled before hitting the floor.

$\vec{s} = \frac{1}{2}(\vec{u} + \vec{v})t$

$= \frac{1}{2}[0 + (-3.9)] \times 0.40$

$= -0.78\,\text{m}$

$\therefore$ height $= +0.78\,\text{m}$

c $\vec{u} = 0\,\text{m}\,\text{s}^{-1}$, $\vec{a} = -9.8\,\text{m}\,\text{s}^{-2}$, $t = 0.20\,\text{s}$, $\vec{s} = ?$

$\vec{s} = \vec{u}t + \frac{1}{2}\vec{a}t^2$

$= 0 + \frac{1}{2} \times -9.8 \times (0.20)^2$

$= 0.20\,\text{m}$ (no direction required for distance)

d The book fell a total of 0.78 m and had fallen 0.20 m in the first 0.2 s.

$0.78 - 0.20 = 0.58\,\text{m}$

6 **a** The time to the top is half of the total time, i.e. 2.0 s.

b $\vec{v} = 0\,\text{m}\,\text{s}^{-1}$, $\vec{a} = -9.8\,\text{m}\,\text{s}^{-2}$, $t = 2\,\text{s}$, $\vec{u} = ?$

$\vec{v} = \vec{u} + \vec{a}t$

$0 = \vec{u} + -9.8 \times 2$

$\vec{u} = 9.8 \times 2$

$= 19.6 = 20\,\text{m}\,\text{s}^{-1}$ (no direction required for speed)

c $\vec{v} = 0$, $\vec{a} = -9.8\,\text{m}\,\text{s}^{-2}$, $t = 2.0\,\text{s}$, $\vec{u} = 19.6\,\text{m}\,\text{s}^{-1}$, $\vec{s} = ?$

$\vec{s} = \vec{u}t + \frac{1}{2}\vec{a}t^2$

$= 19.6 \times 2.0 + \frac{1}{2} \times -9.8 \times 2.0^2$

$= 39.2 - 19.6$

$= +19.6 = 20\,\text{m}$

d The velocity will be the same as its initial velocity but in the opposite direction, i.e. $20\,\text{m}\,\text{s}^{-1}$.

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CHAPTER 2 REVIEW

1 B and D are both scalars. These do not require a direction to be fully described.

2 A and D are vectors. These require a magnitude and direction to be fully described.

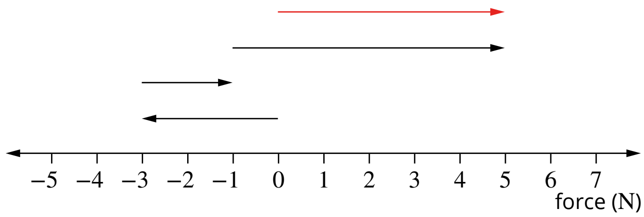
3 The vector must be drawn as an arrow with its tail at the point of contact between the hand and the ball. The arrow points in the direction of the 'push' of the hand.

4 Vector A is drawn twice the length of vector B, so it has twice the magnitude of B.

5 Signs enable mathematical calculations to be carried out on the vectors. Words such as north and south cannot be used in an equation.

6 +80 N or just 80 N

7 The resultant force vector is 5 N.



8 The vectors are (+45.0) + (–70.5) + (+34.5) + (–30.0). This equals –21.0. Backwards is negative, so the answer is 21.0 m backwards.

9 Taking right as positive:

$\Delta\vec{v} = \vec{v} - \vec{u}$

$= -3 + (-3)$

$= -6$

$= 6\,\text{m}\,\text{s}^{-1}$ left

10 To convert $\text{km}\,\text{h}^{-1}$ to $\text{m}\,\text{s}^{-1}$, divide by 3.6:

$\frac{95}{3.6} = 26\,\text{m}\,\text{s}^{-1}$

11 $15\,\text{m}\,\text{s}^{-1} \times 3.6 = 54\,\text{km}\,\text{h}^{-1}$

12 average speed $= \frac{\text{distance}}{\text{time}}$

$= \frac{15\,\text{km}+5\,\text{km}+5\,\text{km}+5\,\text{km}}{2.0}$

$= 15\,\text{km}\,\text{h}^{-1}$

13 **a** average velocity $= \frac{\text{displacement}}{\text{time}}$

$= \frac{20}{2.0}$

$= 10\,\text{km}\,\text{h}^{-1}$ north

b $10\,\text{km}\,\text{h}^{-1}$ north $= \frac{10\,\text{km}\,\text{h}^{-1}\,\text{north}}{3.6}$

$= 2.8\,\text{m}\,\text{s}^{-1}$ north

14 $\Delta v = 4.0 - 6.0$

$= -2.0\,\text{m}\,\text{s}^{-1}$

The change in speed is $-2.0\,\text{m}\,\text{s}^{-1}$. That is, it has decreased by $2.0\,\text{m}\,\text{s}^{-1}$. Speed is a scalar and has no direction.

15 B. The car is moving in a positive direction so its velocity is positive. The car is slowing down so its acceleration is negative.

16 $\vec{a}_{av} = \frac{\vec{v} - \vec{u}}{t}$

$= \frac{-15}{2.5}$

$= -6\,\text{m}\,\text{s}^{-2}$

or

$\vec{u} = 15\,\text{m}\,\text{s}^{-1}$, $\vec{v} = 0\,\text{m}\,\text{s}^{-1}$, $t = 2.5\,\text{s}$, $\vec{a} = ?$

$\vec{v} = \vec{u} + \vec{a}t$

$0 = 15 + \vec{a} \times 2.5$

$\vec{a} = \frac{-15}{2.5} = -6\,\text{m}\,\text{s}^{-2}$

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17 a From 10 to 25 s. (This is the only section of the graph with a positive gradient.)

b From 30 to 45 s. (This is the only section of the graph with a negative gradient.)

c The motorcycle is stationary when the sections on the position–time graph are horizontal. The horizontal sections are from 0 to 10 s, from 25 to 30 s and from 45 to 60 s.

d The motorcycle is passing back through the intersection (at the zero position) at $t = 42.5$ s.

18 a Graph B is the correct answer as it shows speed decreasing to zero to show the car stopping.

b Graph A is the correct graph because it shows a constant value for speed. This is indicated by a straight horizontal line on a velocity–time graph.

c Graph C is the correct graph because it shows velocity increasing from zero in a straight line, indicating uniform acceleration.

19 a Displacement is the area under a velocity–time graph. Area can be determined by counting squares under the graph, then multiplying by the area of each square. This gives approximately 57 squares × (2 × 1) = 114 m.
Alternatively, you can break the area into various shapes and find the sum of their areas:
$72 + 14 + 18 + 10 = 114\,\text{m}$
The result is positive, which means the displacement is north of the starting point.
The cyclist's displacement is 114 m north.

b Average velocity $= \frac{\text{displacement}}{\text{time}}$

$= \frac{114}{11.0}$

$= 10.4\,\text{m}\,\text{s}^{-1}$

c Acceleration is the gradient of the graph. At $t = 1$ s the gradient is flat and therefore zero. This could also be calculated:

gradient $= \frac{\text{rise}}{\text{run}}$

$= 0\,\text{m}\,\text{s}^{-2}$

d Acceleration at t = 10 s is:

gradient $= \frac{\text{rise}}{\text{run}}$

$= -\frac{14}{2}$

$= -7$ or $7\,\text{m}\,\text{s}^{-2}$ south

e A. The velocity is always positive (or zero), indicating that the cyclist travelled only in one direction.

20 $\vec{u} = 0\,\text{m}\,\text{s}^{-1}$, $\vec{a} = 3.5\,\text{m}\,\text{s}^{-2}$, $t = 4.5\,\text{s}$, $\vec{v} = ?$

$\vec{v} = \vec{u} + \vec{a}t$

$= 0 + 3.5 \times 4.5$

$= 15.75$ or $16\,\text{m}\,\text{s}^{-1}$

21 a $\vec{u} = 0\,\text{m}\,\text{s}^{-1}$, $\vec{s} = 2\,\text{m}$, $t = 1\,\text{s}$, $\vec{a} = ?$

$\vec{s} = \vec{u}t + \frac{1}{2}\vec{a}t^2$

$2.0 = 0 + \frac{1}{2} \times \vec{a} \times 1.0^2$

$\vec{a} = 4.0\,\text{m}\,\text{s}^{-2}$

b $\vec{u} = 0\,\text{m}\,\text{s}^{-1}$, $t = 2\,\text{s}$, $\vec{a} = 4.0\,\text{m}\,\text{s}^{-2}$, $\vec{v} = ?$

$\vec{v} = \vec{u} + \vec{a}t$

$= 0 + 4.0 \times 1.0$

$= 4.0\,\text{m}\,\text{s}^{-1}$ (no direction required for speed)

c After 2.0 s the total distance travelled:

$\vec{u} = 0\,\text{m}\,\text{s}^{-1}$, $t = 2\,\text{s}$, $\vec{a} = 4.0\,\text{m}\,\text{s}^{-2}$, $\vec{s} = ?$

$\vec{s} = \vec{u}t + \frac{1}{2}\vec{a}t^2$

$= 0 + 0.5 \times 4.0 \times (2.0)^2$

$= 8.0\,\text{m}$

Distance travelled during the 2nd second is 8.0 m – 2.0 m = 6.0 m.

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22 a $\vec{u} = 10\,\text{m s}^{-1}$, $\vec{v} = 0\,\text{m s}^{-1}$, $\vec{s} = 10\,\text{m}$, $\vec{a} = ?$

$\vec{v}^2 = \vec{u}^2 + 2\vec{a}\vec{s}$

$0 = 10^2 + 2 \times \vec{a} \times 10$

$\vec{a} = -\frac{100}{20}$

$= -5.0\,\text{m s}^{-2}$, forwards (The negative sign in this case is due to deceleration rather than direction).

b $\vec{u} = 10\,\text{m s}^{-1}$, $\vec{v} = 0\,\text{m s}^{-1}$, $\vec{s} = 10\,\text{m}$, $\vec{a} = -5\,\text{m s}^{-2}$, $t = ?$

$\vec{v} = \vec{u} + \vec{a}t$

$0 = 10 - 5t$

$t = 2.0\,\text{s}$

23 a She starts at +4 m.

b She is at rest during sections A and C.

c She is moving in a positive direction during section B with a velocity $+0.8\,\text{m s}^{-1}$.

d She is moving in the negative direction at $2.4\,\text{m s}^{-1}$ during section D.

e $d = 8 + 12$

$= 20\,\text{m}$

$\Delta t = 25\,\text{s}$

$v_{\text{av}} = \frac{d}{\Delta t}$

$= \frac{20}{25}$

$= 0.80\,\text{m s}^{-1}$

24 a $\vec{v} = \vec{u} + \vec{a}t$

$12 = 0 + 1.5t$

$t = 8.0\,\text{s}$

b The bus will catch up with the cyclist when they have each travelled the same distance from the point at which the cyclist first passes the bus.

Cyclist: constant velocity, so $\vec{s} = 12 \times t$

Bus: uniform acceleration $\vec{u} = 0$, $\vec{a} = 1.5\,\text{m s}^{-2}$, $\vec{s} = ?$, $t = ?$

$\vec{s} = \vec{u}t + \frac{1}{2}\vec{a}t^2$

$= 0.75t^2$

When the bus catches up with the cyclist, their displacements are equal, so:

$12t = 0.75t^2$

$t = 16\,\text{s}$

c $\vec{s} = 12 \times 16 = 192\,\text{m}$

25 a Sphere: $\vec{u} = 0\,\text{m s}^{-1}$, $\vec{a} = -9.8\,\text{m s}^{-2}$, $s = -60.0\,\text{m}$, $t = ?$

$\vec{s} = \vec{u}t + \frac{1}{2}\vec{a}t^2$

$-60 = 0 + \frac{1}{2} \times (-9.8) \times t^2$

$t^2 = \frac{-60}{\frac{1}{2} \times (-9.8}$

$t = 3.5\,\text{s}$

b 100 g cube: $\vec{u} = 0.0\,\text{m s}^{-1}$, $\vec{a} = -9.8\,\text{m s}^{-2}$, $s = -70.0\,\text{m}$, $\vec{v} = ?$, $t = ?$

$\vec{v}^2 = \vec{u}^2 + 2\vec{a}\vec{s}$

$= (10.0)^2 + 2 \times (-9.8) \times (-70.0)$

$= 1472$

$\vec{v} = \pm 38.4\,\text{m s}^{-1}$

Because the cube has a downwards velocity, we use the negative value.

$\vec{v} = -38.4\,\text{m s}^{-1}$

$\vec{v} = \vec{u} + \vec{a}t$

$-38.4 = -10.0 - 9.8t$

$9.8t = -10.0 + 38.4$

$t = 2.9\,\text{s}$

You can also solve this using the formula $\vec{s} = \vec{u}t + \frac{1}{2}\vec{a}t^2$ and the quadratic formula.

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26 **a** $\vec{s} = \vec{v}t - \frac{1}{2}\vec{a}t^2$

$15.0 = 0 - 0.5 \times 9.8 \times t^2$

$t = 1.7\,\text{s}$

b From a maximum height of 15.0 m, the ball will fall by 11.0 m. Find how long it takes to travel this distance.

$\vec{s} = \vec{u}t + \frac{1}{2}\vec{a}t^2$

$-11.0 = 0 + 0.5 \times (-9.8) \times t^2$

$t = 1.5\,\text{s}$

Total time from bounce = 1.7 + 1.5 = 3.2 s

27 **a** $v^2 = u^2 + 2\vec{a}\vec{s}$

$= 0 + 2(2.0 \times 4.0)$

$\vec{v} = 4.0\,\text{m s}^{-1}$

b $v^2 = u^2 + 2\vec{a}\vec{s}$

$= 0 + 2(2.0 \times 8.0)$

$\vec{v} = 5.7\,\text{m s}^{-1}$

c $\vec{v} = \vec{u} + \vec{a}t$

$4.0 = 0 + 2.0t$

$t = 2.0\,\text{s}$

d $\vec{v} = \vec{u} + \vec{a}t$

$5.7 = 0 + 2.0t$

$t = 2.85\,\text{s}$

The time taken to travel the final 4.0 m is 2.85 s – 2.0 s = 0.85 s.

28 $\vec{a} = 9.8\,\text{m s}^{-2}$, $t = 0.25\,\text{s}$, $\vec{u}_1 = 0\,\text{m s}^{-1}$

$\vec{s}_1 = \vec{u}_1 t + \frac{1}{2}\vec{a}t^2$

$= 0 + \frac{1}{2} \times 9.8 \times 0.25^2$

$= 0.31\,\text{m}$

The second bolt is 0.31 m away from the first.

$\vec{v}_1 = \vec{u}_1 + \vec{a}t$

$= 0 + 9.8 \times 0.25$

$= 2.45\,\text{m s}^{-1}$

$\vec{a} = 9.8\,\text{m s}^{-2}$, $t = 0.25\,\text{s}$, $\vec{u}_2 = 2.45\,\text{m s}^{-1}$

$\vec{s}_2 = \vec{u}_2 t + \frac{1}{2}\vec{a}t^2$

$= 2.45 \times 0.25 + \frac{1}{2} \times 9.8 \times 0.25^2 = 0.92\,\text{m}$

The third bolt is 0.92 m away from the second.

$\vec{v}_2 = \vec{u}_2 + \vec{a}t$

$= 2.45 + 9.8 \times 0.25 = 4.9\,\text{m s}^{-1}$

$\vec{a} = 9.8\,\text{m s}^{-2}$, $t = 0.25\,\text{s}$, $\vec{u}_3 = 4.9\,\text{m s}^{-1}$

$\vec{s}_3 = \vec{u}_3 t + \frac{1}{2}\vec{a}t^2$

$= 4.9 \times 0.25 + \frac{1}{2} \times 9.8 \times 0.25^2$

$= 1.5\,\text{m}$

The fourth bolt is 1.5 m away from the third.

29 Responses will vary.

The equations of motion are a helpful tool to describe the straight-line motion of an object moving with constant acceleration. In the activity, the object is falling under constant acceleration (gravity), so the time each bolt takes to fall can be predicted. Graphs of velocity versus time and acceleration versus time can be helpful when describing an object's motion.

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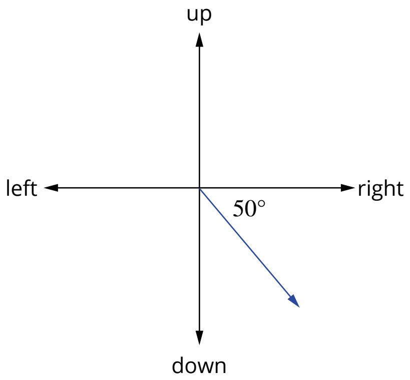
Chapter 3 Motion on a plane

3.1 Vectors in two dimensions

Worked example: Try yourself 3.1.1

DESCRIBING TWO-DIMENSIONAL VECTORS

Describe the direction of the following vector using an appropriate method.

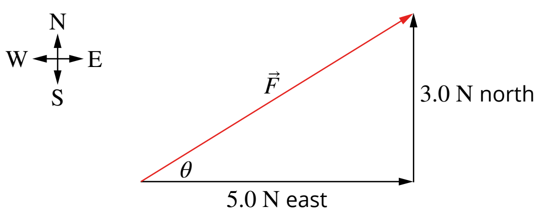


Thinking	Working
Choose the appropriate points to reference the direction of the vector. In this case using the horizontal reference makes more sense, as the angle is given from the horizontal.	The vector can be referenced to the horizontal.
Determine the angle between the reference direction and the vector.	There is 50° from the right direction to the vector.
Determine the direction of the vector from the reference direction.	From the right direction the vector is clockwise.
Describe the vector using the sequence: angle, clockwise or anticlockwise from the reference direction.	This vector is 50° clockwise from the right direction.

Worked example: Try yourself 3.1.2

ADDING VECTORS IN TWO DIMENSIONS USING GEOMETRY

Forces of 5.0 N east and 3.0 N north act on a tree.
Determine the resultant force vector acting on the tree.
Refer to Figure 3.1.3 for sign and direction conventions if required.

Thinking	Working
Construct a vector diagram showing the vectors drawn head to tail. Draw the resultant vector from the tail of the first vector to the head of the last vector.	
As the two vectors to be added are at 90° to each other, apply Pythagoras' theorem to calculate the magnitude of the resultant force.	$F^2 = 5.0^2 + 3.0^2$ $= 25 + 9$ $F = \sqrt{34}$ $= 5.8\,\text{N}$

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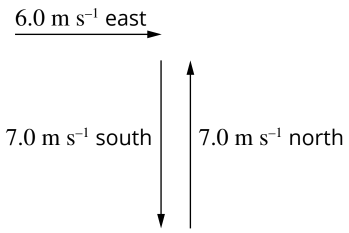
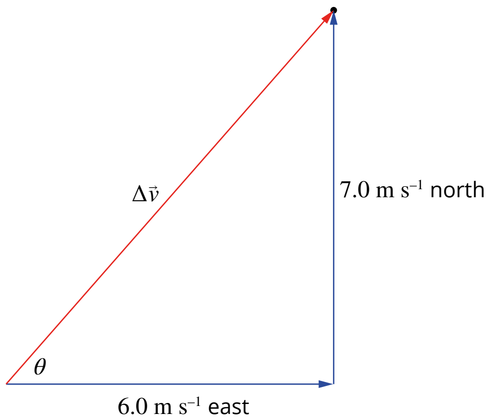
Using trigonometry, calculate the angle from the east vector to the resultant vector.	$\tan\theta = \frac{3.0}{5.0}$ $\theta = \tan^{-1} 0.6$ $= 31.0°$
Determine the direction of the vector relative to north or south.	$90° - 31° = 59°$ The direction is N59°E
State the magnitude and direction of the resultant vector.	$\vec{F} = 5.8\,\text{N}$, N59°E

Worked example: Try yourself 3.1.3

SUBTRACTING VECTORS IN TWO DIMENSIONS USING GEOMETRY

A ball hits a wall at $7.0\,\text{m}\,\text{s}^{-1}$ south and rebounds at $6.0\,\text{m}\,\text{s}^{-1}$ east. Determine the change in velocity of the ball.

Thinking	Working
Draw the final velocity vector $\vec{v}_2$ and the initial velocity vector $\vec{v}_1$ separately. Then draw the initial velocity in the opposite direction.	
Construct a vector diagram, drawing $\vec{v}_2$ first, and then from its head draw the opposite of $\vec{v}_1$. The change of velocity vector is drawn from the tail of the final velocity to the head of the opposite of the initial velocity.	
As the two vectors to be added are at 90° to each other, apply Pythagoras' theorem to calculate the magnitude of the change in velocity.	$v^2 = 7.0^2 + 6.0^2$ $= 49 + 36$ $v = \sqrt{85}$ $= 9.2\,\text{m}\,\text{s}^{-1}$
Calculate the angle from the north vector to the change in velocity vector.	$\tan\theta = \frac{7.0}{6.0}$ $\theta = \tan^{-1} 1.17$ $= 49.4°$ Direction from north vector is $90 - 49.4 = 40.6°$
State the magnitude and direction of the change in velocity.	$\Delta\vec{v} = 9.2\,\text{m}\,\text{s}^{-1}$ N41°E

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3.1 Review

1 $\Delta v^2 = (v_2)^2 + (v_1)^2$

$= 406^2 + 345^2$

$\Delta v = \sqrt{164836 + 119025}$

$= \sqrt{283861}$

$= 533\,\text{m}\,\text{s}^{-1}$

$\tan\theta = \frac{345}{406}$

$\theta = \tan^{-1}\left(\frac{345}{406}\right)$

$= 40.4°$

Angle measured from the north = 90° – 40.4° = 49.6°

$\Delta\vec{v} = 533\,\text{m}\,\text{s}^{-1}$ N49.6°W

2 $\Delta v^2 = (v_2)^2 + (v_1)^2$

$= 42.0^2 + 42.0^2$

$\Delta v = \sqrt{1764 + 1764}$

$= \sqrt{3528}$

$= 59.4\,\text{m}\,\text{s}^{-1}$

$\tan\theta = \frac{42.0}{42.0}$

$\theta = \tan^{-1}(1.000)$

$= 45.0°$

$\Delta\vec{v} = 59.4\,\text{m}\,\text{s}^{-1}$ N45.0°W

3 $\Delta v^2 = (v_2)^2 + (v_1)^2$

$= 5.25^2 + 7.05^2$

$\Delta v = \sqrt{(27.56 + 49.70)}$

$= \sqrt{77.27}$

$= 8.79\text{ m s}^{-1}$

$\tan\theta = \frac{7.05}{5.25}$

$\theta = \tan^{-1}\left(\frac{7.05}{5.25}\right)$

$= 53.3°$

Angle measured from the north = 90° – 53.3° = 36.7°

$\Delta\vec{v} = 8.79\,\text{m}\,\text{s}^{-1}$ N36.7°W

4 $R^2 = 40^2 + 20^2$

$= 1600 + 400$

$R = \sqrt{2000}$

$= 44.7 = 45\,\text{m}$

$\tan\theta = \frac{40.0}{20.0}$

$\theta = \tan^{-1} 2.00$

$= 63°$

$\vec{R} = 45\,\text{m}$, S63°W

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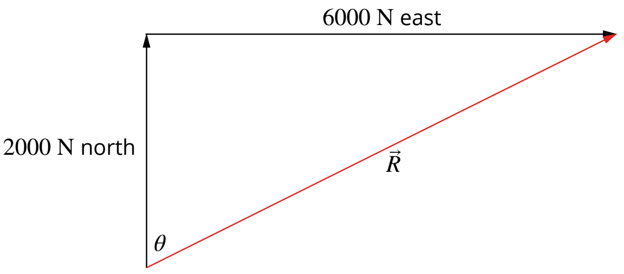
5



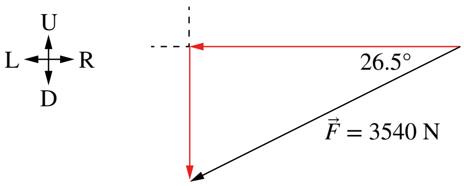
$$R^2 = 2000^2 + 6000^2$$
$$= 4\,000\,000 + 36\,000\,000$$
$$R = \sqrt{40\,000\,000}$$
$$= 6325\,\text{N}$$
$$\tan\theta = \frac{6000}{2000}$$
$$\theta = \tan^{-1} 3.00$$
$$= 71.57°$$
$$\vec{R} = 6325\,\text{N, N}71.57°\text{E}$$

3.2 Vector components

Worked example: Try yourself 3.2.1

CALCULATING THE PERPENDICULAR COMPONENTS OF A FORCE

A 3540 N force acts on a trolley in a direction 26.5° anticlockwise from the left direction.
Use the direction conventions to determine the perpendicular components of the force.

Thinking	Working
Draw $\vec{F}_L$ from the tail of the 3540 N force along the horizontal, then draw $\vec{F}_D$ from the horizontal line to the head of the 3540 N force.	
Calculate the left component of the force $\vec{F}_L$ using $\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}}$.	$\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}}$ $\text{adj} = \text{hyp} \times \cos\theta$ $\vec{F}_L = 3540 \times \cos 26.5°$ $= 3168\,\text{N left}$
Calculate the downwards component of the force $\vec{F}_D$ using $\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}}$.	$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}}$ $\text{opp} = \text{hyp} \times \sin\theta$ $\vec{F}_D = 3540 \times \sin 26.5°$ $= 1580\,\text{N downwards}$

3.2 Review

1 a $\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}}$

$$\text{opp} = \text{hyp} \times \sin\theta$$
$$\vec{F}_D = 462 \times \sin 35.0°$$
$$= 265\,\text{N downwards}$$

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b $\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}}$

$\text{adj} = \text{hyp} \times \cos\theta$

$\vec{F}_R = 462 \times \cos 35.0°$

$= 378\,\text{N right}$

2 $\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}}$

$\text{adj} = \text{hyp} \times \cos\theta$

$\vec{F}_S = 25.9 \times \cos 40.0°$

$= 19.8\,\text{N south}$

$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}}$

$\text{opp} = \text{hyp} \times \sin\theta$

$\vec{F}_E = (25.9)(\sin 40.0°)$

$= 16.6\,\text{N east}$

So 19.8 N south and 16.6 N east.

3 $\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}}$

$\text{adj} = \text{hyp} \times \cos\theta$

$\vec{v}_N = 18.3 \times \cos 75.6°$

$= 4.55\,\text{m s}^{-1}\ \text{north}$

$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}}$

$\text{opp} = \text{hyp} \times \sin\theta$

$\vec{v}_W = 18.3 \times \sin 75.6°$

$= 17.7\,\text{m s}^{-1}\ \text{west}$

So $4.55\,\text{m s}^{-1}$ north and $17.7\,\text{m s}^{-1}$ west.

4 $\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}}$

$\text{ad} = \text{hyp} \times \cos\theta$

$\vec{s}_S = (47.0)(\cos 66.3°)$

$= 18.9\,\text{m south}$

$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}}$

$\text{opp} = \text{hyp} \times \sin\theta$

$\vec{s}_S = 47.0 \times \sin 66.3°$

$= 43.0\,\text{m east}$

Zehn is 18.9 m south and 43 m east of his starting point.

5 $\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}}$

$\text{adj} = \text{hyp} \times \cos\theta$

$\vec{F}_N = 235\,000 \times \cos 62.5°$

$= 109\,000\,\text{N north}$

$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}}$

$\text{opp} = \text{hyp} \times \sin\theta$

$\vec{F}_W = 235\,000 \times \sin 62.5°$

$= 208\,450\,\text{N west}$

6 **a** $\vec{F}_S = 100 \cos 60° = 50\,\text{N south}$

$\vec{F}_E = 100 \sin 60° = 87\,\text{N east}$

b $\vec{F}_N = 60\,\text{N north}$

c $\vec{F}_S = 300 \cos 20° = 282\,\text{N south}$

$\vec{F}_E = 300 \sin 20° = 103\,\text{N east}$

d $\vec{F}_v = 3.0 \times 10^5 \sin 30° = 1.0 \times 10^5\,\text{N upwards}$

$\vec{F}_h = 3.0 \times 10^5 \cos 30° = 2.6 \times 10^5\,\text{N horizontally}$

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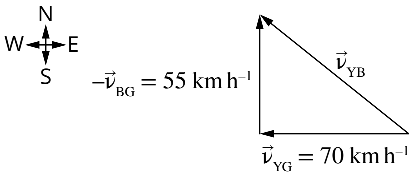
3.3 Relative motion

Worked example: Try yourself 3.3.1

FIND THE RELATIVE VELOCITY BETWEEN TWO CARS

A black car and a yellow car are travelling down the same road towards the south at 55 km h^{-1}. The yellow car turns off onto a side road towards the west and travels at 70 km h^{-1}.

Find the velocity of the yellow car as it travels on the side road, relative to the black.

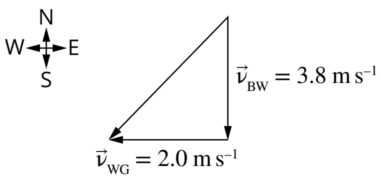
Thinking	Working
Define your vectors with appropriate notation. Write out the equation for the velocity of the blue car relative to the silver car.	$\vec{v}_{YG}$ = velocity of the yellow car relative to the ground $\vec{v}_{BG}$ = velocity of the black car relative to the ground $\vec{v}_{YB}$ = velocity of the yellow car relative to the black car $\vec{v}_{YB} = \vec{v}_{YG} + \vec{v}_{GB}$ $= \vec{v}_{YG} + (-\vec{v}_{BG})$
Construct a vector diagram showing the vectors drawn head to tail. Draw the resultant vector from the tail of the first vector to the head of the last vector.	
As the two vectors to be added are at 90° to each other, apply Pythagoras' theorem to calculate the magnitude of the resultant velocity.	$v_{YB}^2 = 70^2 + 55^2$ $= 4900 + 3025$ $v_{YB} = \sqrt{7925}$ $= 89$ km h^{-1}
Using trigonometry, calculate the angle from the east vector to the resultant vector.	$\tan\theta = \frac{70}{55}$ $\theta = \tan^{-1}(1.3)$ $= 51.8° = 52°$
State the magnitude and direction of the resultant vector.	$\vec{v}_{YB} = 89$ km h^{-1} N52°W

Worked example: Try yourself 3.3.2

CALCULATE RELATIVE VELOCITY OF A BOAT ON A RIVER

A boat is travelling at 3.8 m s^{-1} south across a river relative to the water. The current is flowing upstream at 2.0 m s^{-1} west.

Determine the velocity of the boat relative to the ground, giving the direction to the nearest degree.

Thinking	Working
Write out the equation describing the resultant velocity.	$\vec{v}_{BG} = \vec{v}_{BW} + \vec{v}_{WG}$
Construct a vector diagram showing the vectors drawn head to tail. Draw the resultant vector from the tail of the first vector to the head of the last vector.	
As the two vectors to be added are at 90° to each other, apply Pythagoras' theorem to calculate the magnitude of the resultant velocity.	$v_{BG}^2 = 3.8^2 + 2.0^2$ $= 14.44 + 4.0$ $v_{BG} = \sqrt{18.44}$ $= 4.3$ m s^{-1}

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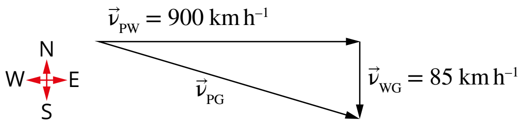
Using trigonometry, calculate the angle from the west vector to the resultant vector.	$\tan\theta = \frac{2.0}{3.8}$ $\theta = \tan^{-1} 0.5$ $= 28°$
Determine the direction of the vector relative to north or south.	The direction is S28°W.
State the magnitude and direction of the resultant vector.	$\vec{v}_{BG} = 4.3\,\text{m}\,\text{s}^{-1}$, S28°W

Worked example: Try yourself 3.3.3

FIND THE RESULTANT VELOCITY OF AN AEROPLANE IN A CROSS WIND

A jet aircraft is travelling $900\,\text{km}\,\text{h}^{-1}$ east, with a crosswind blowing at $85.0\,\text{km}\,\text{h}^{-1}$ south. Determine the velocity of the plane relative to the ground.

Thinking	Working
Write out the equation describing the resultant velocity.	$\vec{v}_{PG} = \vec{v}_{PW} + \vec{v}_{WG}$
Construct a vector diagram showing the vectors drawn head to tail. Draw the resultant vector from the tail of the first vector to the head of the last vector.	
As the two vectors to be added are at 90° to each other, apply Pythagoras' theorem to calculate the magnitude of the resultant velocity.	$v_{PG}^2 = 900^2 + 85^2$ $= 810\,000 + 7225$ $v_{PG} = \sqrt{817\,225}$ $= 904\,\text{km}\,\text{h}^{-1}$
Using trigonometry, calculate the angle from the west vector to the resultant vector.	$\tan\theta = \frac{85.0}{900}$ $\theta = \tan^{-1} 0.09$ $= 5.4°$
Determine the direction of the vector relative to north or south.	$90° - 5.4° = 84.6°$ The direction is S84.6°E
State the magnitude and direction of the resultant vector.	$\vec{v}_{PG} = 904\,\text{km}\,\text{h}^{-1}$, S84.6°E

3.3 Review

1 $\vec{v}_{CT} = \vec{v}_{CG} + \vec{v}_{GT}$

$= \vec{v}_{CG} - \vec{v}_{TG}$

2 $v^2 = 0.8^2 + 2.5^2$

$= 0.64 + 6.25$

$v = \sqrt{6.98}$

$= 2.6\,\text{m}\,\text{s}^{-1}$

$\tan\theta = \frac{0.8}{2.5}$

$\theta = \tan^{-1} 0.32$

$= 17.7°$

$\vec{v} = 2.6\,\text{m}\,\text{s}^{-1}$ N17.7°W

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3 $\vec{v}_{BP} = \vec{v}_{BW} + \vec{v}_{WP}$

$= \vec{v}_{BW} + (-\vec{v}_{PW})$

First calculate the magnitude of the resultant vector:

$v_{BP}{}^2 = 5^2 + 2^2$

$v_{BP} = \sqrt{29}$

$= 5.4\,m\,s^{-1}$

Calculate the angle of the resultant velocity:

$\tan\theta = \frac{5.0}{2.0}$

$\theta = \tan^{-1} 2.5$

$= 68.2° = 68°$

$\vec{v}_{BP} = 5.4\,m\,s^{-1}$ N68°E

4 $\tan\theta = \frac{50}{300}$

$\theta = \tan^{-1} 0.167$

$=$ N9.46°E

5 **a** $\vec{v}_{PG}$ = velocity of the plane relative to the ground

$\vec{v}_{WG}$ = velocity of the wind relative to the ground

$\vec{v}_{PW}$ = velocity of the plane relative to the wind

$\vec{v}_{PG} = \vec{v}_{PW} + \vec{v}_{WG}$

$= 910 - 70$

$= 840\,km\,h^{-1}$ south

b Calculate the magnitude of the resultant velocity

$v_{PG}{}^2 = 910^2 + 70^2$

$= \sqrt{833\ 000}$

$= 913\ km\ h^{-1}$

Calculate the angle of the resultant velocity

$\tan\theta = \frac{70}{910}$

$\theta = \tan^{-1} 0.08$

$= 4.40°$

$\vec{v}_{PG} = 913\,km\,h^{-1}$ S4.40°E

6 $\vec{v}_{21} = \vec{v}_{2G} + \vec{v}_{G1}$

$= \vec{v}_{2G} + (-\vec{v}_{1G})$

$= 319 - 315$

$= 4\,km\,h^{-1}$ towards the finish line

7 $\vec{v}_{ES} = \vec{v}_{EG} + \vec{v}_{GS}$

$= \vec{v}_{EG} + (-\vec{v}_{SG})$

$v^2 = 6^2 + 20^2$

$= 36 + 400$

$v = \sqrt{436}$

$= 20.9\ m\ s^{-1}$

$\tan\theta = \frac{6}{20}$

$\theta = \tan^{-1} 0.3$

$= 16.7°$

Calculate the direction from either north or south.

$90° - 16.7° = 73.3°$

$\vec{v}_{ES} = 21\,m\,s^{-1}$, N73°W

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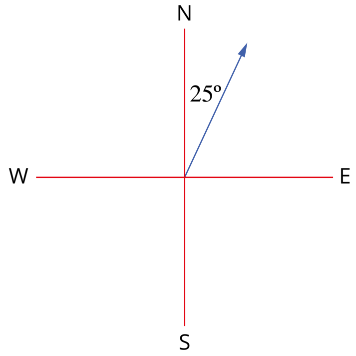
CHAPTER 3 REVIEW

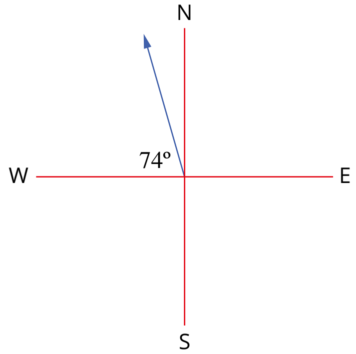
1 $34.0\,m\,s^{-1}$ north and $12.5\,m\,s^{-1}$ east. This is because the change in velocity is the final velocity plus the opposite of the initial velocity. The opposite of $34.0\,m\,s^{-1}$ south is $34.0\,m\,s^{-1}$ north.

2 70° anticlockwise from the left.

3 **a**



b



4 **a** sin - the opposite and hypotenuse edges are given.

b sin - the opposite and hypotenuse edges are given.

c tan - the opposite and adjacent edges are given.

d cos - the adjacent and hypotenuse edges are given.

5 $s^2 = 7.2^2 + 4.5^2$

$= 51.84 + 20.25$

$s = \sqrt{72.09}$

$= 8.5\,m$

This is the magnitude of the displacement; therefore it doesn't need a direction.

6 $\vec{v}_W = 6.5 \times \sin 22°$

$= 2.4\,m\,s^{-1}$ west

$\vec{v}_N = 6.5 \times \cos 22°$

$= 6.0\,m\,s^{-1}$ north

7 C

$R^2 = 40.0^2 + 30.0^2$

$= 1600 + 900$

$R = \sqrt{2500}$

$= 50.0\,m$

8 horizontal component $\vec{F}_N = 300 \cos 60° = 150\,N$

vertical component $\vec{F}_V = 300 \sin 60° = 260\,N$

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9 C. Distance is the length that has been travelled, in this case 300 + 400 = 700 km. Displacement is a vector, and is the shortest distance from the beginning of the journey to the end, given with a direction. The journey of the plane forms two sides of a right-angled triangle, with the hypotenuse being the displacement, hence 500 km. The direction is given by the angle between the hypotenuse and due north, or:

$\tan^{-1}\frac{400}{300}$

Or instead imagine the aeroplane going 400 km east along the *x*-axis and then 300 km north to give an angle of 36.9 degrees. The direction will then be:

$90 - 36.9 = \text{N}53.1°\text{E}$

10 $\vec{v}_H = 5.2 \cos 30°$

$= 4.5\ \text{m s}^{-1}$ horizontally

$\vec{v}_V = 5.2 \sin 30°$

$= 2.6\ \text{m s}^{-1}$ vertically

11 Vertical component of the first half:

$s_1 = 8.0 \sin 20°$

$= 2.7\ \text{m}$

Vertical component of the second half:

$s_2 = 8.0 \sin 30°$

$= 4.0\ \text{m}$

Height of the slope = 2.7 + 4.0 = 6.7 m

12 $\vec{v}_{JG} = \vec{v}_{JW} + \vec{v}_{WG}$

Calculate the magnitude of the resultant vector:

$v^2 = 0.3^2 + 0.8^2$

$= 0.09 + 0.64$

$v = \sqrt{0.73}$

$= 0.85\ \text{m s}^{-1}$

Calculate the angle of the resultant vector.

$\tan\theta = \frac{0.3}{0.8}$

$\theta = \tan^{-1} 0.375$

$= 20.56°$

$90 - 20.56 = \text{N}69.4°\text{E}$

$\vec{v}_{JG} = 0.85\ \text{m s}^{-1}\ \text{N}69°\text{E}$

13 $\vec{v}_{PH} = \vec{v}_{PG} + \vec{v}_{GH}$

$= \vec{v}_{PG} - \vec{v}_{HG}$

14 $\vec{v}_{WG} = 300 \sin 8°$

$= 42\ \text{km h}^{-1}$

15 $\vec{v}_{YG} = \vec{v}_{YW} + \vec{v}_{WG}$

Calculate the magnitude of the resultant vector:

$v^2 = 8.0^2 + 2.2^2$

$= 64 + 4.84$

$v = \sqrt{68.84}$

$= 8.3\ \text{m s}^{-1}$

Calculate the angle of the resultant vector.

$\tan\theta = \frac{2.2}{8.0}$

$\theta = \tan^{-1} 0.275$

$= 15.4°$

$\vec{v}_{TC} = 8.3\ \text{m s}^{-1}\ \text{N}15°\text{W}$

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16 a North. A tail wind blows in the same direction of motion as the plane.

b $\vec{v}_{PG} = \vec{v}_{PW} + \vec{v}_{WG}$

$= 890 + 40$

$= 930\,\text{km h}^{-1}$ north

17 $\vec{v}_{BG} = \vec{v}_{BW} + \vec{v}_{WG}$

Calculate the magnitude of the resultant vector.

$v^2 = 2.4^2 + 3.0^2$

$= 5.76 + 9.0$

$v = \sqrt{14.76}$

$= 3.8\,\text{m s}^{-1}$

Calculate the angle of the resultant vector.

$\tan\theta = \frac{2.4}{3.0}$

$\theta = \tan^{-1} 0.80$

$= 38.7° = 39°$

$\vec{v}_{BG} = 3.8\,\text{m s}^{-1}$, N39°W

18 $\vec{v}_{TC} = \vec{v}_{TG} + \vec{v}_{GC}$

$= \vec{v}_{TG} + (-\vec{v}_{CG})$

Calculate the magnitude of the resultant vector.

$v^2 = 100^2 + 60^2$

$= 10\,000 + 3600$

$v = \sqrt{13\,600}$

$= 116.6 = 120\,\text{km h}^{-1}$

Calculate the angle of the resultant vector.

$\tan\theta = \frac{60}{100}$

$\theta = \tan^{-1} 0.60$

$= 30.96°$

$90 - 30.96 =$ N59°E

$\vec{v}_{TC} = 120\,\text{km h}^{-1}$, N59°E

19 a $\vec{v}_E = 0.8 \sin 10$

$= 0.14\,\text{m s}^{-1}$ east

$\vec{v}_N = 0.8 \cos 10$

$= 0.79\,\text{m s}^{-1}$ north

b The vector components show the relative velocity of the wind to the table and the marble to the wind. i.e. They describe the equation:

$\vec{v}_{MT} = \vec{v}_{MW} + \vec{v}_{W}$

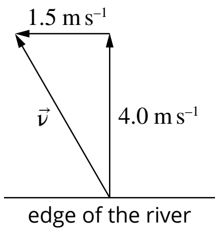
where

$\vec{v}_{MT}$ is the velocity of the marble relative to the table

$\vec{v}_{MW}$ is the velocity of the marble relative to the wind

$\vec{v}_{WT}$ is the velocity of the wind relative to the table

20 a



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b $v^2 = 4^2 + 1.5^2$

$= 16 + 2.25$

$v = \sqrt{18.25}$

$= 4.3\,\text{m}\,\text{s}^{-1}$

$\tan\theta = \frac{1.5}{4.0}$

$\theta = \tan^{-1}(0.375)$

$= 20.56°$

Calculate the angle from the edge of the river.

Angle = 90 – 20.56

= 69.4° = 69°

$\vec{v} = 4.3\,\text{m}\,\text{s}^{-1}$ at 69° from the edge of the river

c Convert five minutes into seconds.

$5 \times 60 = 300\,\text{s}$

$\vec{v}_{av} = \frac{\vec{s}}{\Delta t}$

$\vec{s} = \vec{v}_{av}\Delta t$

$= 4.27 \times 300$

$= 1282 = 1300\,\text{m}$ at 69° from the edge of the river

d To find the width of the river, you need to find the components of the displacement vector.

$\vec{s}_x$ = distance boat travelled downstream

$\vec{s}_y$ = width of the river

$\cos\theta = \frac{\vec{s}_y}{\vec{s}}$

$\cos(20.56) = \frac{\vec{s}_y}{1282}$

$\vec{s}_y = 0.93 \times 1282$

$= 1200\,\text{m}$ in the forward direction of the boat

e $\sin\theta = \frac{\vec{s}_x}{\vec{s}}$

$\sin(20.56) = \frac{\vec{s}_x}{1282}$

$\vec{s}_y = 0.35 \times 1282$

$= 450\,\text{m}$ downstream

21 a A ball thrown through the air experiences three force vectors that affect its motion. These are the forward force from the initial throw, the force due to air resistance, and the downwards weight force due to gravity. The horizontal and vertical motion is slowed by air resistance, and the weight force pulls the ball down.

b The ball is a projectile; that is, after it is launched into the air the only forces that act on it are air resistance and gravity. All projectile motion can be described by a parabola.

22 Responses will vary.

Motion in two and three dimensions can be analysed by considering the perpendicular components independently. In this experiment the original motion (along the table) and the direction of the fan (across the table) are perpendicular.

The fan provides a force on the marble across the table, causing an acceleration in the direction of the force. The component of the velocity in this direction increases. This is similar to gravitational acceleration in the vertical component of projectile motion.

The component along the table should remain relatively unchanged. There will be some friction present, which will slow the motion in this direction. This is similar to the horizontal component of projectile motion.

Each component can be analysed using the equations of motion to predict how an object's velocity and displacement will change over time.

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Module 1 Review answers

Kinematics

MULTIPLE CHOICE

1 D. Applying the equations of motion:

$\vec{u} = 0$, $\vec{a} = 5.5\,\text{m s}^{-2}$, $t = 3\,\text{s}$

$$\vec{s} = \vec{u}t + \tfrac{1}{2}\vec{a}t^2$$

$$= 0 + \tfrac{1}{2} \times 5.5 \times 3^2$$

$s = 24.75\,\text{m}$

2 A. The distance travelled in the first three seconds (s_3) minus the distance travelled in the first two seconds (s_2) is the distance travelled in the third second of motion.

Finding the distance travelled in the first two seconds:

Applying the equations of motion:

$\vec{u} = 0$, $\vec{a} = 2.5\,\text{m s}^{-2}$, t = between 2 and 3 seconds

$$\vec{s}_2 = \vec{u}t + \tfrac{1}{2}\vec{a}t^2$$

$$= 0 + \tfrac{1}{2} \times 2.5 \times 2^2$$

$s_2 = 5\,\text{m}$

$$\vec{s}_3 = \vec{u}t + \tfrac{1}{2}\vec{a}t^2$$

$$= 0 + \tfrac{1}{2} \times 2.5 \times 3^2$$

$s_3 = 11.25\,\text{m}$

$\therefore s_3 - s_2 = 11.25 - 5 = 6.25\,\text{m}$

3 **a** C. Distance is found by taking the magnitude of the area under the graph.

The area is found by using the formula for the area of a trapezium.

$$\text{Area} = \tfrac{1}{2}(a + b)h$$

$$= \tfrac{1}{2}(2 + 6) \times 0.2$$

$$= 0.8\,\text{m}$$

b B. Displacement is found by calculating the area under the graph, taking care to consider whether the area is positive (moving away from the starting point) or negative (moving towards the starting point).

The area of the positive section was found in the previous question (+0.8 m).

The area of the 'negative' section found by using the formula for the area of a trapezium.

$$\text{Area} = \tfrac{1}{2}(a + b)h$$

$$= \tfrac{1}{2}(3 + 5) \times -0.1$$

$$= -0.4\,\text{m}$$

Displacement = +0.8 + (–0.4)

= +0.4 m or 0.4 m east, as train was initially moving east.

4 B.

$\vec{u} = 0,\quad \vec{v} = \vec{v}_1,\quad \vec{s} = h,\quad \vec{a} = \vec{g}$

$\vec{v}_1^{\,2} = \vec{u}^2 + 2\vec{a}\,\vec{s} = 0 + 2 \times \vec{g} \times h$

$\vec{v}_1 = \sqrt{2\vec{g}h}$

$\vec{u} = 0,\quad \vec{v} = \vec{v}_2,\quad \vec{s} = 2h,\quad \vec{a} = \vec{g}$

$\vec{v}_2^{\,2} = \vec{u}^2 + 2\vec{a}\,\vec{s} = 0 + 2 \times \vec{g} \times 2h$

$\vec{v}_2 = \sqrt{4\vec{g}h} = \sqrt{2\vec{v}_1}$

5 A. Change in velocity $= \vec{v}_{\text{final}} - \vec{v}_{\text{initial}}$

$= (-3) - (+5) = -8\,\text{m s}^{-1}$

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6 C. Distance is the length that has been travelled, in this case 300 + 400 = 700 km. Displacement is a vector, and is the shortest distance from the beginning of the journey to the end, given with a direction. The journey of the plane forms two sides of a right-angled triangle, with the hypotenuse being the displacement, hence 500 km. The direction is given by the angle between the hypotenuse and due north, or $\tan^{-1}\left(\frac{400}{300}\right)$.
Or instead imagine the aeroplane going 400 km east along the *x*-axis and then 300 km north to give an angle of 36.9 degrees. The direction will then be
90 – 36.9 = N53.1°E

7 C.
$v^2 = u^2 + 2as$
$v^2 = 0 + 2 \times 2 \times 100$
$v = 20\,\text{m s}^{-1}$
$= 72\,\text{km h}^{-1}$

8 D.
$\vec{v}_{BS} = \vec{v}_{BG} + \vec{v}_{GS} = \vec{v}_{BG} + (-\vec{v}_{SG})$

9 **a** C. The distance of the race = area under the *v*–*t* graph from $t = 0$ s to $t = 11$ s is $d = 100$ m.
b B. Total distance = area under the graph from $t = 0$ s to $t = 15$ s is s = 120 m.
Average speed $= \frac{\text{distance}}{\text{time}} = \frac{120}{15} = 8.0\,\text{m s}^{-1}$

10 **a** A, B, C: those sections of the graph with a positive gradient
b A, E: those sections of the graph with a non-zero gradient whose magnitude is increasing with respect to time.

11 C.
$\Delta v^2 = (v_2)^2 + (-v_1)^2$
$= (2.1)^2 + (6.3)^2$
$\Delta v = \sqrt{4.41 + 39.69}$
$= \sqrt{44.1}$
$= 6.6\,\text{m s}^{-1}$

12 D.
$\Delta v^2 = (v_2)^2 + (-v_1)^2$
$= (7.2)^2 + (6.9)^2$
$\Delta v = \sqrt{51.84 + 47.61}$
$= \sqrt{99.45}$
$= 9.97\,\text{m s}^{-1}$
$\tan\theta = \frac{6.9}{7.2}$
$\theta = \tan^{-1}(0.958)$
$= 43.8°$
$\Delta v = 10\,\text{m s}^{-1}$ N44°W

13 B.
$v_S = 100 \times \cos 44 = 72\,\text{km h}^{-1}$
$v_W = 100 \times \sin 44 = 70\,\text{km h}^{-1}$

14 A.
$v_N = 6 \times \cos 12 = 5.9\,\text{m s}^{-1}$
$v_E = 6 \times \sin 12 = 1.2\,\text{m s}^{-1}$

15 D.
$\vec{v}_{JG} = \vec{v}_{JW} + \vec{v}_{WG}$
${v_{JG}}^2 = 1.3^2 + 1.0^2$
$= 1.69 + 1.0$
$v = \sqrt{2.69}$
$= 1.64\,\text{m s}^{-1}$
Calculate the angle of the resultant vector.
$\tan\theta = \frac{1.0}{1.3}$
$\theta = \tan^{-1} 0.77$
$= 37.6°$
$=$ N37.6°E
$\vec{v}_{JG} = 1.6\,\text{m s}^{-1}$, N38°E

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16 C.

$\vec{v}_{MT} = \vec{v}_{MG} + \vec{v}_{GT} = \vec{v}_{MG} + (-\vec{v}_{TG})$

$\vec{v}_{MT} = 60 + 150$

$= +210$

$= 210\,\text{km h}^{-1}$ east

17 D.

$\vec{v}_{PW} = 600\,\text{km h}^{-1}$ south

$\vec{v}_{WG} = 38\,\text{km h}^{-1}$ north

$\vec{v}_{PG} = \vec{v}_{PW} + \vec{v}_{WG}$

$= 600 - 38$

$= 562\,\text{km h}^{-1}$ south

distance = speed × time

$= 562 \times 2$

$= 1124\,\text{km}$

18 A.

v_{av} = distance ÷ time

$= 6000\,\text{m} \div (2 \times 60 \times 60\,\text{s})$

$= 0.8\,\text{m s}^{-1}$

19 A.

$\vec{v}_{BG} = 5.5\,\text{m s}^{-1}$ east

$\vec{v}_{MG} = 5.1\,\text{m s}^{-1}$ north

$\vec{v}_{BM} = \vec{v}_{BG} + \vec{v}_{GM}$

$\vec{v}_{BM} = \vec{v}_{BG} + (-\vec{v}_{MG})$

= 5.5 east + 5.1 south

$v_{BM}{}^2 = 5.5^2 + 5.1^2$

$v_{BM} = \sqrt{30.25 + 26.01}$

$= 7.5\,\text{m s}^{-1}$

20 C.

$\vec{v}_{PW} = 102\,\text{km h}^{-1}$ south

$\vec{v}_{WG} = 30\,\text{km h}^{-1}$ south

$\vec{v}_{PG} = \vec{v}_{PW} + \vec{v}_{WG}$

$= 102 + 30$

$= 132\,\text{km h}^{-1}$ south

SHORT ANSWER

21 **a** When the arrow reaches its maximum height its vertical velocity = 0.

$\vec{v} = \vec{u} + \vec{a}t$

$0 = 100 - 9.8t$

$t = 10.2\,\text{s}$

b $\vec{s} = \vec{u}t + \frac{1}{2}\vec{a}t^2$

$= 100 \times 10.2 + \frac{1}{2} - (9.8\,\text{m s}^{-2})(10.2\,\text{s}^2)$

$= 510\,\text{m}$

c The acceleration of the arrow is constant during its flight and is equal to $9.8\,\text{m s}^{-2}$ downwards.

22 **a** Displacement north = $8 + 7\sin 45° = 12.95\,\text{km}$

Displacement east = $7\cos 45° = 4.95\,\text{km}$

Total displacement = 13.9 km on a bearing of $90 - \tan^{-1}\left(\frac{12.95}{4.95}\right) = 21° =$ N21°E

b Average speed is calculated using distance travelled rather than the displacement. Speed $= \frac{(8+7)\times 10^3\,\text{m}}{7 \times 60 \times 60\,\text{s}} = 0.6\,\text{m s}^{-1}$

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23 a $\vec{s} = \vec{u}t + \frac{1}{2}\vec{a}t^2$

Therefore $-122 = -4.9t^2$ so $t = 5$ s

b $\vec{s} = \vec{u}t + \frac{1}{2}\vec{a}t^2$

Therefore $-122 = -10t - 4.9t^2$, $t = 4.07$ s
(alternatively calculate v using $v^2 = u^2 + 2as$ and then use $v = u + at$ to find the time)

c platinum sphere: $\frac{122}{5} = 24.4\,\text{m s}^{-1}$, lead sphere: $\frac{122}{4.07} = 30.0\,\text{m s}^{-1}$ downwards

24 a For the first 30 s the cyclist travels east 150 m at a constant speed. Then they accelerate for the next 10 s, travelling 150 m. They then travel at a higher constant speed for the next 10 s, travelling another 200 m.

b gradient = 150 ÷ 30 = 5 m s^{-1} east

c gradient = 200 ÷ 10 = 20 m s^{-1} east

d $\vec{v}_{av} = \frac{\vec{s}}{t} = \frac{150}{10} = 15\,\text{m s}^{-1}$ east

e $\vec{a} = \frac{\vec{v} - \vec{u}}{t} = 1.5\,\text{m s}^{-2}$ east

f $v_{\text{av}} = \frac{d}{t} = \frac{500}{50} = 10\,\text{m s}^{-1}$

25 a Convert the velocity to m s^{-1}: 90 km h^{-1} = 25 m s^{-1}; 60 km h^{-1} = 16.67 m s^{-1}

$a = \frac{(16.61\,\text{ms}^{-1} - 25\,\text{ms}^{-1})}{12\,\text{s}}$

$= -0.69\,\text{m s}^{-2}$

= deceleration of 0.69 m s^{-2}

b $v^2 = u^2 + 2as$

$16.67^2 = 25^2 - 2 \times 0.69 \times s$

$\therefore s = 2.5 \times 10^2$ m

26 a $\vec{s} = \frac{1}{2}(\vec{u} + \vec{v})t$

$t = \frac{2\vec{s}}{\vec{u} + \vec{v}} = \frac{2 \times 20}{15 + 0} = 2.7$ s

b $\vec{v} = \vec{u} + \vec{a}t$

$\vec{a} = \frac{\vec{v} - \vec{u}}{t} = \frac{0 - 15}{2.7} = -5.6\,\text{m s}^{-2}$

27 a $72\,\text{km h}^{-1} = \frac{72}{3.6} = 20\,\text{m s}^{-1}$

$d = vt$

$= 20 \times 0.50 = 10$ m

b $v = u + at$

$0 = 20 + (-4.0)t$

$t = 5.0$ s

c Reaction distance: $d = 10$ m

Braking distance: $s = ut + \frac{1}{2}at^2$

$s = 20 \times 5.0 + \frac{1}{2}(-4.0) \times (5.0)^2 = 50$ m

Total stopping distance: 10 m + 50 m = 60 m

28 a From the starting point (the origin), the person walks forward in a positive direction for 2.5 metres in 2 seconds, and then stays in that position for 4 more seconds. Then they turn and walk back in a negative direction, passing the origin, to 2.5 metres behind the starting point over 2 seconds. They stay at this point for 2 seconds before walking back to the origin.

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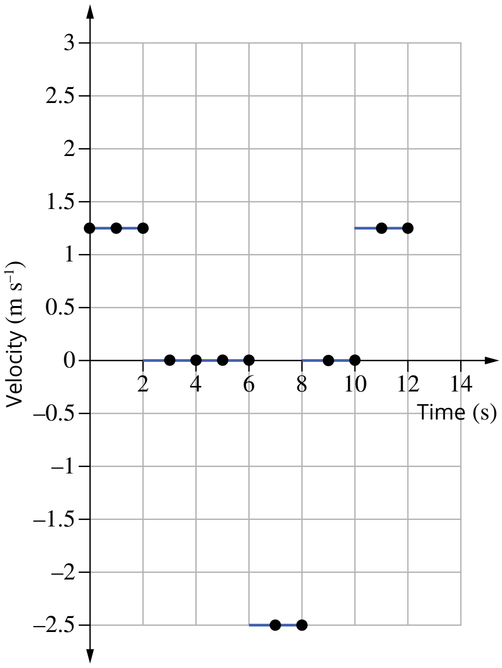
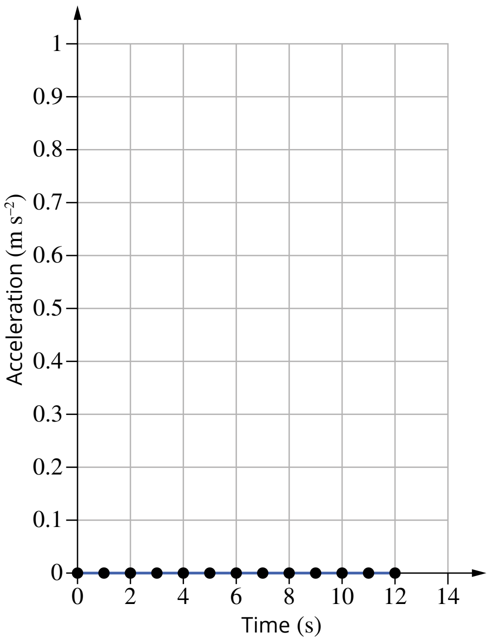
b



c



Although accelerations (or decelerations) must occur at 0, 2, 6, 8, 10 and 12 seconds because the velocity changes at these times, they are more or less instantaneous and therefore cannot be shown on the graph.

29 a $v_{av} = \frac{d}{\Delta t} = \frac{100}{9.58} = 10.4\,\text{m}\,\text{s}^{-1}$

b $\Delta t = \frac{d}{v_{av}} = \frac{1000}{10.44} = 95.8\,\text{s} = 1.6$ minutes

30 a $\vec{s} = \vec{u}t + \frac{1}{2}\vec{a}t^2$

$10 = 0 + \frac{1}{2} \times 9.8 \times t^2$

$t^2 = \frac{20}{9.8} = 2.04$

$t = 1.4\,\text{s}$

b $\vec{v} = \vec{u} + \vec{a}t$

$= 0 + 9.8 \times 1.4$

$= 13.72 = 14\,\text{m}\,\text{s}^{-1}$

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EXTENDED RESPONSE

31 **a** **i** $v_{av} = \frac{d}{t} = \frac{1\times10^{-2}}{0.10} = 0.10\,m\,s^{-1}$

ii $v_{av} = \frac{d}{t} = \frac{3.0\times10^{-2}}{0.10} = 0.30\,m\,s^{-1}$

iii $v_{av} = \frac{d}{t} = \frac{5.0\times10^{-2}}{0.10} = 0.50\,m\,s^{-1}$

b The instantaneous speeds of the marble in the centre of the time interval are equal to the average speed over the entire interval.

i $v = 0.10\,m\,s^{-1}$

ii $v = 0.30\,m\,s^{-1}$

iii $v = 0.50\,m\,s^{-1}$

c The marble is moving with constant acceleration.

32 **a** $v_{ANZ}{}^2 = v_{AW}{}^2 + v_{NZW}{}^2$

$= 26^2 + 22^2$

$v_{ANZ} = \sqrt{1160}$

$= 34\,km\,h^{-1}$

b Divide $km\,h^{-1}$ by 3.6 to convert to $m\,s^{-1}$:

$\vec{s}_A = (26 \div 3.6) \times 60$

$= 433\,m$ south

$\vec{s}_{NZ} = (22 \div 3.6) \times 60$

$= 367\,m$ east

$s^2 = 433^2 + 367^2$

$= 322222.18$

$s = 567.6\,m$

c **i** $1.5 \times 3.6 = 5.4\,km\,h^{-1}$

$\vec{v}_{NZG} = \vec{v}_{NZW} + \vec{v}_{WG}$

$v_{NZG}{}^2 = v_{NZW}{}^2 + v_{WG}{}^2$

$= 22^2 + 5.4^2$

$v_{NZG} = \sqrt{513.1}$

$= 22.65\,km\,h^{-1}$

$\tan\theta = \frac{5.4}{22.0}$

$\theta = \tan^{-1} 0.245$

$= 13.8°$

$90 - 13.8 = $ S76.2°E

$\vec{v}_{NZG} = 22.65\,km\,h^{-1}$, S76.2°E

ii $\vec{v}_{AG} = \vec{v}_{AW} + \vec{v}_{WG}$

$= 26 + 5.4$

$\vec{v}_{AG} = 31.4\,km\,h^{-1}$ south

d $\vec{s}_A = (31.4 \div 3.6\,m\,s^{-1}) \times (60\,s) + 433\,m$

$= 956.67\,m$ south

$\vec{s}_{NZ} = (22.65 \div 3.6\,m\,s^{-1}) \times (60\,s)$ S76.2°E $+ 367\,m$ east

$= 377.5$ S76.2°E $+ 367\,m$ east

Find the vector components:

$\vec{s}_E = 377.5 \times \sin 76.2 + 367 = 733.6\,m$ east

$\vec{s}_S = 377.5 \times \cos 76.2 = 90.05\,m$ south

$s^2 = (956.67 - 90.05)^2 + 733.6^2$

$= 1288626.7$

$s = 1135\,m = 1.1\,km$

33 **a** $\vec{v}_{TG} = \vec{v}_{TW} + \vec{v}_{WG} = \vec{v}_{TW} + (-\vec{v}_{GW})$

$v^2 = 3.6^2 + 2.5^2$

$v = \sqrt{19.21}$

$= 4.38 = 4.4\,m\,s^{-1}$

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$\tan\theta = \frac{3.6}{2.5}$

$\theta = \tan^{-1} 1.44$

$= 55.2° = 55°$

$90 - 55 = N35°E$

$\vec{v}_{TG} = 4.4\,m\,s^{-1}$, N35°E

b Break $\vec{v}_{TW}$ into its north and east components. Relative to the ground, Tessa is now on a bearing of $3.1 - 2.5 = 0.6\,m\,s^{-1}$ west and still at $3.6\,ms^{-1}$ north. The magnitude of her velocity relative to the ground is $\sqrt{3.6^2 + 0.6^2} = 3.65\,m\,s^{-1}$ and the bearing is $\theta = \tan^{-1}\frac{3.6}{0.6} = 80.5°$ measured clockwise from the horizontal axis, so her velocity is $3.65\,m\,s^{-1}$ N 9.5°W.

c Her displacement will be $3.65 \times 60 = 219\,m$ on the same bearing, N9.5°W

d Yes, up to rounding error.

34 a $\vec{a} = \frac{\Delta\vec{v}}{t}$

$= \frac{\vec{v} - \vec{u}}{t}$

$\vec{v} - \vec{u} = \vec{a}t$

$\vec{v} = \vec{u} + \vec{a}t$

b $\vec{v}_{av} = \frac{1}{2}(\vec{v} + \vec{u})$

$\vec{v} = \frac{1}{2}((\vec{u} + at) + \vec{u})$

$= \frac{1}{2}(2\vec{u} + \vec{a}t)$

Multiply by time

$\vec{v}t = \vec{u}t + \frac{1}{2}\vec{a}t^2$

Substitue $\vec{s} = \vec{v}t$ into the equation

$\vec{s} = \vec{u}t + \frac{1}{2}\vec{a}t^2$

c $\vec{s} = \vec{u}t + \frac{1}{2}\vec{a}t^2$

Substitute in for t:

$\vec{a} = \frac{\vec{v} - \vec{u}}{t} \quad t = \frac{\vec{v} - \vec{u}}{\vec{a}}$

$\vec{s} = \vec{u}\left(\frac{\vec{v} - \vec{u}}{\vec{a}}\right) + \frac{1}{2}\vec{a}\left(\frac{\vec{v} - \vec{u}}{\vec{a}}\right)^2$

$= \frac{\vec{u}\vec{v} - \vec{u}^2}{\vec{a}} + \frac{\vec{v}^2 - 2\vec{u}\vec{v} + \vec{u}^2}{2\vec{a}}$

$2\vec{a}\vec{s} = 2(\vec{u}\vec{v} - \vec{u}^2) + \vec{v}^2 - 2\vec{u}\vec{v} + \vec{u}^2$

$= -\vec{u}^2 + \vec{v}^2$

$\vec{v}^2 = \vec{u}^2 + 2\vec{a}\vec{s}$

35 a $\vec{v}_{21} = \vec{v}_{2G} + \vec{v}_{G1} = \vec{v}_{2G} + (-\vec{v}_{1G})$

Break each of these into their components

$\vec{v}_{1N} = 50 \times \sin 45 = 35.355$

$\vec{v}_{1E} = 50 \times \cos 45 = 35.355$

$\vec{v}_{2N} = 60 \times \sin 10 = 10.419$

$\vec{v}_{2E} = 60 \times \cos 10 = 59.088$

$v_{21}{}^2 = (35.36 + 10.42)^2 + (35.36 + 59.09)^2$

$= 45.77^2 + 94.4^2$

$v_{21} = 104.95 = 105\,km\,h^{-1}$

$\tan\theta = \frac{45.7}{94.4}$

$\theta = \tan^{-1} 0.48$

$= 25.8°$

$90 - 25.8 = 64.2 = N64°E$

$\vec{v}_{21} = 105\,km\,h^{-1}$, N64°E

b $\vec{v}_{21} = -\vec{v}_{12}$

$\vec{v}_{12} = 105\,km\,h^{-1}$, S64°W

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c $\vec{s}_1 = \vec{v}_1 t = 50 \times 1.5 = 75\,\text{km}$ southwest
$\vec{s}_2 = \vec{v}_2 t = 60 \times 1.5 = 90\,\text{km}$ N80°E
Break these into their components
$\vec{s}_{1S} = 75 \times \cos 45 = 53.033$
$\vec{s}_{1W} = 75 \times \sin 45 = 53.033$
$\vec{s}_{2S} = 90 \times \sin 10 = 15.628$
$\vec{s}_{2W} = 90 \times \cos 10 = 88.633$
$s^2 = (53.03 + 15.63)^2 + (53.03 + 88.63)^2$
$= 68.66^2 + 141.67^2$
$s = 157.4 = 157\,\text{km}$

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Chapter 4 Forces

4.1 Newton's first law

Worked example: Try yourself 4.1.1

CALCULATING NET FORCE AND EQUILIBRIUM IN ONE DIMENSION

Consider two forces acting on an object. A 20 N force acts on the object towards the left, and a 23 N force acts on the object towards the right.

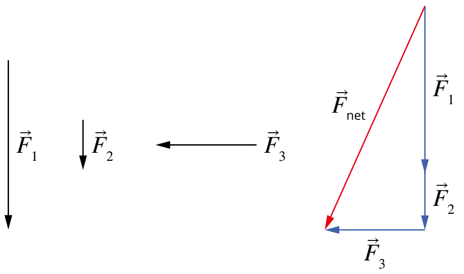
Calculate the net force acting on the object, and determine what additional force is required for the object to be in equilibrium.

Thinking	Working
Determine the individual forces acting on the object.	$\vec{F}_1 = 20\,\text{N left}$ $\vec{F}_2 = 23\,\text{N right}$
Apply a sign convention to replace directions.	$\vec{F}_1 = -20\,\text{N}$ $\vec{F}_2 = +23\,\text{N}$
Determine the net force acting on the object.	$\vec{F}_{net} = \vec{F}_1 + \vec{F}_2$ $= (-20) + 23$ $= 3\,\text{N right}$
Determine what additional force is required for the object to be in equilibrium.	Equilibrium exists when $\Sigma\vec{F} = 0$ $\vec{F}_{net} = 3\,\text{N right}$ For equilibrium to exist, a force of 3 N towards the left would be required to act on the object.

Worked example: Try yourself 4.1.2

CALCULATING NET FORCE AND EQUILIBRUM IN TWO DIMENSIONS

Consider three forces acting on an object: a 10 N downwards force, a 2 N downwards force, and a 5 N force from right to left.

a Calculate the net force acting on the object.	
Thinking	**Working**
Determine the individual forces acting on the object.	$\vec{F}_1 = 10\,\text{N down}$ $\vec{F}_2 = 2\,\text{N down}$ $\vec{F}_3 = 5\,\text{N left}$
Apply a sign convention to replace directions.	Choose down and left to be positive directions. This means up and right are represented as negative.
Create a vector diagram describing the net force acting on the object.	

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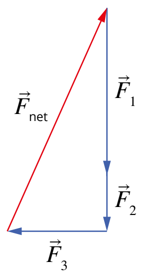
Calculate the magnitude of the net force using Pythagoras' theorem.	$\vec{F}_{net} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3$ $\vec{F}_{net}$ = 10 N (down) + 2 N (down) + 5 N (left) $\vec{F}_{net}$ = 12 N (down) + 5 N (left) $\vec{F}_{net}{}^2 = 12^2 + 5^2$ $\vec{F}_{net} = \sqrt{169}$ $\vec{F}_{net}$ = 13 N
Determine the angle of the net force.	$\tan\theta = \frac{5}{12}$ $\theta = \tan^{-1} 0.42$ = 22.6° in the left direction from the vertical
State the net force.	$\vec{F}_{net}$ = 13 N at an angle of 22.6° in the left direction from the vertical

b Determine what additional force is required for the object to be in equilibrium.	
Thinking	**Working**
Determine what additional force is required for the object to be in equilibrium.	Equilibrium exists when: $\Sigma\vec{F} = 0$ $\vec{F}_{net}$ = 12 N downwards and 5 N towards the left So for equilibrium to exist, an additional force of 12 N upwards and 5 N towards the right must act on the object. This vector is equal to the negative value of $\vec{F}_{net}$ calculated in part **a**. 

4.1 Review

1 The box has changed its velocity, so the student can use Newton's first law to conclude that an unbalanced force must have acted on the box to slow it down.

2 Even though the car has maintained its speed, the direction has changed, which means the velocity has changed. From Newton's first law, it can be concluded that an unbalanced force has acted on the car to change its direction.

3 B. Because the ball maintains a constant velocity, according to Newton's first law there must not be an unbalanced force. There is no forwards force, friction or air resistance acting on the ball.

4 No force pushes them forward. In accordance with Newton's first law of motion, when the bus stops suddenly the passenger will continue to move with constant velocity unless acted on by an unbalanced force; they will probably lose their footing and fall forwards.

5 The aeroplane slows down very rapidly as it travels along the runway because of the large retarding forces acting on it. The passengers wearing seatbelts would have retarding forces provided by the seatbelt and would slow down at the same rate as the plane. A passenger standing in the aisle, if they were not hanging on to anything, would have no retarding forces acting and so would tend to maintain their original velocity and move rapidly towards the front of the aeroplane.

6 **a** The inertia of the glass makes it want to remain at rest. Because the cloth is pulled quickly, the force of friction between the cloth and the glass acts only for a very short time. This time is not long enough to enable the friction force to overcome the inertia of the glass and make it move.

b Using a full glass makes the trick easier because the inertia of a full glass is greater than that of an empty glass.

7 The speed and altitude are constant, so the net force must be zero in both vertical and horizontal directions. To exactly balance the other forces, lift must be 50 kN upwards, and drag must be 12 kN west.

8 $\vec{F}_{net} = \vec{F}_1 + \vec{F}_2$

= 20 + (–15)

= 5 N to the right

Equilibrium exists when the vector sum of all forces acting on an object result in a zero net force. To maintain equilibrium, a third force of 5 N must act on the object to the left.

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4.2 Newton's second law

Worked example: Try yourself 4.2.1

CALCULATING THE FORCE THAT CAUSES AN ACCELERATION

Calculate the net force on a 75.8 kg runner who is accelerating at 4.05 m s^{-2} south.

Thinking	Working
Ensure that the variables are in their standard units.	$m = 75.8$ kg $\vec{a} = 4.05$ m s^{-2} south
Apply the equation for force from Newton's second law.	$\vec{F} = m\vec{a}$ $= 75.8 \times 4.05$ $= 307$ N
Give the direction of the net force, which is the same as the direction of the acceleration.	$\vec{F} = 307$ N south

Worked example: Try yourself 4.2.2

CALCULATING THE ACCELERATION OF AN OBJECT WITH MORE THAN ONE FORCE ACTING ON IT

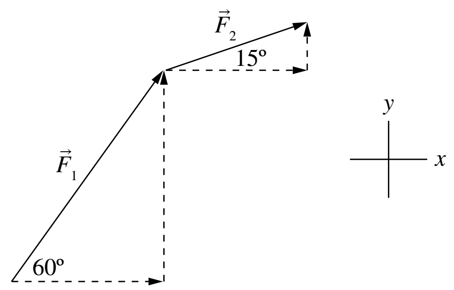
A car with a mass of 900 kg applies a driving force of 3000 N forwards as it starts moving. Friction and air resistance oppose the motion of the car with a force of 750 N backwards.
What is the car's initial acceleration?

Thinking	Working
Determine the individual forces acting on the car, and apply the vector sign convention.	$\vec{F}_1 = 3000$ N forwards $= 3000$ N $\vec{F}_2 = 750$ N backwards $= -750$ N
Determine the net force acting on the car.	$\vec{F}_{net} = \vec{F}_1 + \vec{F}_2$ $= 3000 + (-750)$ $= +2250$ N or 2250 N forwards
Use Newton's second law to determine acceleration.	$\vec{a} = \frac{\vec{F}_{net}}{m}$ $= \frac{2250}{900}$ $= 2.50$ m s^{-2} forwards

Worked example: Try yourself 4.2.3

CALCULATING THE ACCELERATION OF AN OBJECT WITH A TWO-DIMENSIONAL FORCE ACTING ON IT

A 150 N force acts at an angle of 60° to the x direction on an object with a mass of 75 kg. A second force of 80 N acts on the same object at an angle of 15° to the x direction.



What is the net force and initial acceleration acting on the object in the y direction?

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Thinking	Working
Determine the x and y components of the force acting on the object.	$\vec{F}_1 = 150\,\text{N at } 60°$ $\vec{F}_2 = 80\,\text{N at } 15°$ $\vec{F}_{1(y)} = 150 \sin 60°$ $= 129.9\,\text{N}$ $\vec{F}_{2(y)} = 80 \sin 15°$ $= 20.7\,\text{N}$
Determine the net force acting on the object in the y direction.	$\vec{F}_{\text{net}} = \vec{F}_{1y} + \vec{F}_{2y}$ $= 129.9 + 20.7$ $= 150.6\,\text{N}$ upwards
Use Newton's second law to determine acceleration.	$\vec{a} = \frac{\vec{F}_{\text{net}}}{m}$ $= \frac{150.6}{75.0}$ $= 2.0\,\text{m s}^{-2}$ upwards

4.2 Review

1 $\vec{a} = \frac{\vec{F}_{\text{net}}}{m}$

$= \frac{158}{23.9}$

$= 6.61\,\text{m s}^{-2}$ north

2 $\vec{a} = \frac{\vec{F}_{\text{net}}}{m}$

$= \frac{441}{45.0}$

$= 9.80\,\text{m s}^{-2}$ down

3 $\vec{a} = \frac{\vec{F}_{\text{net}}}{m}$

$= \frac{882}{90.0}$

$= 9.80\,\text{m s}^{-2}$ down

4 $m = \frac{\vec{F}_{\text{net}}}{\vec{a}}$

$= \frac{565\,000}{7.20}$

$= 78\,500\,\text{kg}$

5 **a** $\vec{F}_g = m\vec{g} = 15 \times 9.8 = 147\,\text{N}$ down

b $\vec{F}_{\text{net}} = 10\,\text{N north} + 8.5\,\text{N south}$

$= 10 + (-8.5)$

$= 1.5$

$= 1.5\,\text{N}$ north

c $\vec{F}_{\text{net}} = m\vec{a}$

$1.5 = 15\vec{a}$

$\vec{a} = 0.1\,\text{m s}^{-2}$ north

d If the model is moving at a constant speed, the net force on the model must be zero, so the force of the breeze must be equal and opposite to the total drag forces, i.e. 7.2 N north.

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6 Determine the driving force provided by the truck based on the information for when it is empty.

$\vec{F} = m\vec{a}$

$= 2000 \times 2.0$

$= 4000\,\text{N}$

Calculate the total mass of the truck for the new acceleration.

$m = \frac{\vec{F}_{\text{net}}}{\vec{a}}$

$= \frac{4000}{1.25}$

$= 3200\,\text{kg}$

So the mass of the boxes must be:

$3200 - 2000 = 1200$ kg.

Number of boxes $= \frac{1200}{300}$

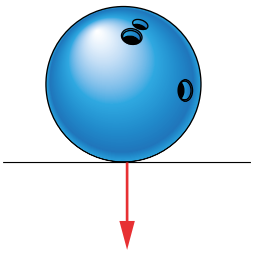
$= 4$ boxes

4.3 Newton's third law

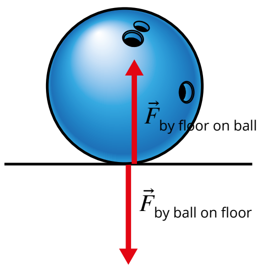
Worked example: Try yourself 4.3.1

APPLYING NEWTON'S THIRD LAW

In the diagram below, a bowling ball is resting on the floor, and the action force is shown.



a Identify the action and reaction forces using the system $\vec{F}_{\text{by ... on ...}}$	
Thinking	**Working**
Identify the two objects involved in the action–reaction pair.	The bowling ball and the floor.
Identify which object is applying the force and which object is experiencing the force, for the force vector shown.	The action force vector shown is a force by the bowling ball on the floor.
Use the system of labelling action and reaction forces to label the action and reaction forces.	Action force: $\vec{F}_{\text{by ball on floor}}$ Reaction force: $\vec{F}_{\text{by floor on ball}}$

b Draw the reaction force on the diagram, showing its size and location, and label both forces.	
Copy the diagram into your workbook. Use a ruler to measure the length of the action force and construct a vector arrow in the opposite direction with its tail on the point of application of the reaction force. Label the forces.	

4.3 Review

1 There is a force on the hammer by the nail, and a force on the nail by the hammer. These two forces are equal in magnitude and opposite in direction.

2 **a** The force arrow shown is the $\vec{F}_{\text{by the Earth on the astronaut}}$

b The reaction force must act on the other object, so in this case it is the $\vec{F}_{\text{by the astronaut on the Earth}}$

3 The force on the hand by the water. The swimmer creates the action force by pushing on the water, but the reaction force acts on the swimmer which moves her in the direction of her motion.

4 **a** The boat exerts an equal and opposite reaction force, i.e. 140 N in the opposite direction to the leaping fisherman.

b $m = \frac{\vec{F}_{net}}{\vec{a}}$

$= \frac{140}{40}$

$= 3.5\,\text{m s}^{-2}$ in the opposite direction to the fisherman

c Acceleration of the fisherman: $\vec{a} = \frac{\vec{F}_{net}}{m} = \frac{140}{70} = 2.0\,\text{m s}^{-2}$

Speed of the fisherman: $v = 0 + 2 \times 0.5 = 1.0\,\text{m s}^{-1}$

Speed of the boat: $v = 0 + 3.5 \times 0.5 = 1.8\,\text{m s}^{-1}$

5 The astronaut should throw the tool kit away from the ship. By throwing the tool kit, there is an action force on the tool kit by the astronaut which is directed away from the ship. According to Newton's third law, there will be a reaction force on the astronaut by the tool kit that will be in the opposite direction, i.e. towards the ship.

6 Tania is correct. For an action–reaction pair, the action force is a force on object A by object B, and the reaction force is a force on object B by object A. That is, the two forces act on different objects. In this case, both the weight force and the normal force are acting on the same object: the lunch box.

7 $\vec{F}_g = m\vec{g}$

$= 10 \times 9.8$

$= 98\,\text{N}$

$\vec{F}_N = m\vec{g}\cos\theta$

$= 10 \times 9.8\cos 60°$

$= 49\,\text{N}$ perpendicular to the plane

CHAPTER 4 REVIEW

1 A and C, because the forces act directly on the objects. B and D are examples of non-contact forces, or forces mediated by a field.

2 No, a force has not pushed the passengers backwards. The passengers have inertia, so their masses resist the change in motion as the train starts moving forwards. According to Newton's first law, their bodies are simply maintaining their original state of being motionless until an unbalanced force acts to accelerate them.

3 C. An object travelling at a constant velocity will do so when the net force is equal to zero.

4 **a** $\Sigma\vec{F} = \vec{F}_1 + \vec{F}_2$

$= 10 + (-5)$

$= 5\,\text{N}$ to the right.

b No. The forces are not in equilibrium as the net force is not equal to zero.

5 An internal force has no effect on the motion of the object, whereas an external force does. If you are sitting in your car, pushing against the steering wheel is an example of an internal force in the car. Internal forces will always be resisted by another part of the object, leading to equilibrium. If a bus pushes the car, that is an example of an external force acting on the car.

6 $\vec{F}_x = 50\cos 45°$

$= 35\,\text{N}$ in the positive x direction

$\vec{F}_y = 50\sin 45°$

$= 35\,\text{N}$ in the positive y direction

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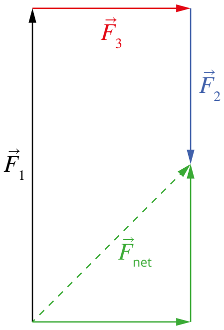
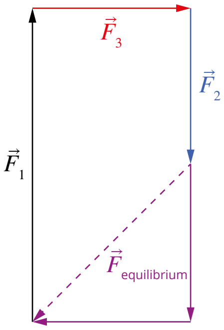
7 **a** $\vec{F}_1$ is the 20 N upwards force, $\vec{F}_2$ is the 10 N downwards force, and $\vec{F}_3$ is the 10 N force from left to right. $\vec{F}_{net}$ is the resultant force (or net force).



b $\vec{F}_1$ is the 20 N upwards force, $\vec{F}_2$ is the 10 N downwards force, and $\vec{F}_3$ is the 10 N force from left to right. $\vec{F}_{equilibrium}$ is the force required for the object to be in equilibrium.



8 B. If the same force (magnitude and direction) is applied to two equal masses, the acceleration will be the same and in the same direction.

9 $\vec{a} = \frac{\vec{F}_{net}}{m}$

$= \frac{352}{9.20}$

$= 38.3\ kg$

10 $\vec{a} = \frac{\vec{F}_{net}}{m}$

$= \frac{95.0}{0.0609}$

$= 1560\ m\,s^{-2}$ south

11 $\vec{a} = \frac{\vec{F}_{net}}{m}$

$= \frac{3550}{657}$

$= 5.40\ m\,s^{-2}$ north

12 $\vec{F}_1 = 150\ N$ at 45°

$\vec{F}_2 = 15\ N$ at 30°

$\vec{F}_{1(x)} = 150 \cos 45°$

$= 106\ N$ in the positive x direction

$\vec{F}_{2(x)} = 15 \cos 30°$

$= 13\ N$ in the positive x direction

$\Sigma\vec{F}_x = \vec{F}_{1(x)} + \vec{F}_{2(x)}$

$= 106 + 13$

$= 119\ N$ in the positive x direction

$\vec{a} = \frac{\vec{F}_{net}}{m}$

$= \frac{119}{10}$

$= 11.9\ m\,s^{-2}$ in the positive x direction

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13 $\vec{F}_1 = 10\,\text{N}$ at 60°

$\vec{F}_2 = 40\,\text{N}$ at 15°

$\vec{F}_{1(y)} = 10 \sin 60°$

$= 8.66$ N in the positive y direction

$\vec{F}_{2(y)} = 40 \sin 15°$

$= 10.4\,\text{N}$ in the positive y direction

$\Sigma\vec{F}_y = \vec{F}_{1(y)} + \vec{F}_{2(y)}$

$= 8.66 + 10.4$

$= 19\,\text{N}$ in the positive y direction

$\vec{a} = \frac{\vec{F}_{\text{net}}}{m}$

$= \frac{19}{75}$

$= 0.25\,\text{m}\,\text{s}^{-2}$ in the positive y direction

14 The bus will accelerate more slowly than the motorcycle. Newton's second law states that an object's acceleration is inversely proportional to the object's mass (i.e. $\vec{a} \propto \frac{1}{m}$). Therefore, given the same force, a lighter object will accelerate faster than a heavier object.

15 $\vec{F}_g = m\vec{g}$

$\vec{F}_g = 7 \times 9.8$

$= 69\,\text{N}$

$\vec{F}_\text{N} = m\vec{g} \cos\theta$

$= 7 \times 9.8 \cos 65°$

$= 29\,\text{N}$ (The magnitude of the force is a scalar so no direction is required.)

16 $\vec{F}_{\text{net}}$ = thrust – weight of rocket

$= 1\,000\,000 - 50\,000 \times 9.8$ (remembering that $1000\,\text{kN} = 1\,000\,000\,\text{N}$)

$= 510\,000\,\text{N}$

$\vec{a} = \frac{\vec{F}_{\text{net}}}{m}$

$= \frac{510\,000}{50\,000}$

$= 10.2\,\text{m}\,\text{s}^{-2}$, upwards

17 C. If a person jumps upwards, their legs push down on the Earth. This is the action force. The reaction force is the Earth pushing up on their legs.

18 The force on the balloon comes from the escaping air. The balloon's elasticity compresses the air inside and pushes it out of the mouth of the balloon. This is the action force. The air must therefore exert an equal and opposite forwards force on the balloon, which in turn moves the balloon around the room.

19 The reaction force is on the racquet by the ball, resulting in a force of 100 N east.

20 Newton's third law states that every action has an equal and opposite reaction. Therefore the reaction of the board acting on the skateboarder results in a force of 75.0 N north.

21 Responses will vary.

Forces are broken down into contact forces and forces mediated by a field. Contact forces occur when two objects come in contact with each other, e.g. a tennis racquet hitting a ball.

Non-contact forces act on object within a field, e.g. the gravitational force keeping the Moon in orbit around the Earth.

According to Newton's first law, an object will remain at rest unless an unbalanced force acts on it. An object at rest (or moving at constant velocity) will be in equilibrium, so the sum of all forces acting on it are equal to zero. Using the concept of equilibrium and Newton's second law ($\vec{F} = m\vec{a}$) it is possible to predict the motion of an object by analysing the forces acting upon it.

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Chapter 5 Forces, acceleration and energy

5.1 Forces and friction

Worked example: Try yourself 5.1.1

CALCULATING FRICTION

A person pushes a wardrobe weighing 100 N from one room to another. Calculate the force of friction applied to the wardrobe if the coefficient of kinetic friction between the wardrobe and the floor is 0.5.

Thinking	Working
Recall the definition of friction.	friction = $\mu\vec{F}_N$
The normal force will be equal and opposite to the weight of the box.	$\vec{F}_N$ = 100 N upwards
Substitute in the values for this situation to find the kinetic friction.	$\vec{F}_k = 0.5 \times 100$
Solve the problem, giving an answer with appropriate units and direction.	$\vec{F}_k$ = 50 N in the opposite direction to the motion of the wardrobe

Worked example: Try yourself 5.1.2

DETERMINING THE EFFECT OF FRICTION

An air hockey puck slides in a straight line across an air hockey table with negligible friction. If the air stopped blowing so that the puck was in contact with the table, what would happen to the motion of the puck?

Thinking	Working
Recall Newton's first law of motion and the concept of inertia.	An object will maintain a constant velocity unless an unbalanced, external force acts on it.
Identify the motion of the puck.	Without friction, the puck travels in a straight line at a constant speed.
Identify the effect of friction when the air stops blowing.	Friction will cause the puck to decelerate.
Determine the effect on the puck's motion.	The puck will slow down and stop sliding.

Worked example: Try yourself 5.1.3

CALCULATING DECELERATION DUE TO FRICTION

An air hockey puck with a mass of 100 g slides across an air hockey table. The coefficient of kinetic friction is 0.5.

a Calculate the force of friction between the puck and the table.	
Thinking	Working
Recall the definition of friction.	friction = $\mu\vec{F}_N$
The normal force will be equal and opposite to the weight of the puck.	$\vec{F}_g = m\vec{g}$ $= 0.1 \times 9.8$ = 0.98 N down $\vec{F}_N$ = 0.98 up
Substitute in the values for this situation into the definition of friction to find the kinetic friction.	$\vec{F}_k = 0.5 \times 0.98$
Solve the problem, giving an answer with appropriate units and direction.	$\vec{F}_k$ = 0.49 N in the opposite direction to the motion of the puck

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b Calculate the acceleration of the puck caused by friction.	
Thinking	**Working**
Recall Newton's second law.	$\vec{F} = m\vec{a}$
Transpose the formula to make acceleration the subject and solve for $\vec{a}$.	$\vec{a} = \frac{\vec{F}}{m}$ $= \frac{0.49}{0.1}$ $= 4.9\,\text{m s}^{-2}$
Determine the acceleration.	$\vec{a} = 4.9\,\text{m s}^{-2}$ in the opposite direction to the puck's motion.

Worked example: Try yourself 5.1.4

CALCULATING THE ACCELERATION OF A CONNECTED BODY WITH AND WITHOUT FRICTION

A 0.6 kg trolley cart is connected by a cord to a 1.5 kg mass. The cord is placed over a pulley and allowed to fall under the influence of gravity.

a Assuming that there is no friction between the cart and the table and the pulley is frictionless, determine the acceleration of the cart.	
Thinking	**Working**
Recognise that the cart and the falling mass are connected, and determine a sign convention for the motion.	As the mass falls, the cart will move forwards. Therefore, both downwards movement of the mass and forwards movement of the cart will be considered positive motion.
Write down the data that is given. Apply the sign convention to vectors.	$m_1 = 1.5\,\text{kg}$ $m_2 = 0.6\,\text{kg}$ $\vec{g} = 9.8\,\text{m s}^{-2}$ down $= +9.8\,\text{m s}^{-2}$
Determine the forces acting on the system.	The only force acting on the combined system of the cart and mass is the weight of the falling mass. $\vec{F}_{net} = m_1\vec{g}$ $= 1.5 \times 9.8$ $= 14.7\,\text{N}$ in the positive direction
Calculate the total mass being accelerated.	This net force has to accelerate not only the cart but also the falling mass. $m = m_1 + m_2$ $= 1.5 + 0.6$ $= 2.1\,\text{kg}$
Use Newton's second law to determine the acceleration of the cart.	$\vec{a} = \frac{\vec{F}_{net}}{m}$ $= \frac{14.7}{2.1}$ $= 7.0\,\text{m s}^{-2}$ forwards

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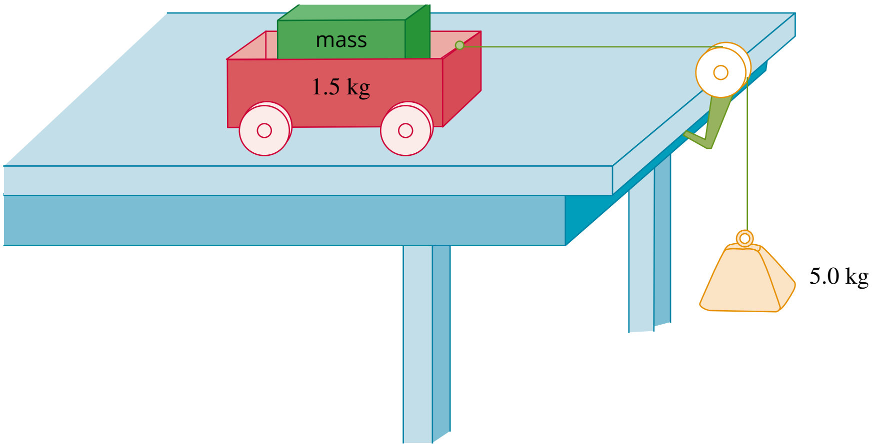
b If a frictional force of 4.2 N acts against the cart, what is the acceleration now?	
Thinking	**Working**
Write down the data that is given. Apply the sign convention to vectors.	$m_1 = 1.5\,\text{kg}$ $m_2 = 0.6\,\text{kg}$ $\vec{g} = 9.8\,\text{m s}^{-2}$ down $= +9.8\,\text{m s}^{-2}$ friction = 4.2 N backwards $= -4.2\,\text{N}$
Determine the forces acting on the system.	There are now two forces acting on the combined system of the cart and mass: the weight of the falling mass and friction. $\vec{F}_{\text{net}} = \vec{F}_g + \text{friction}$ $= 14.7 + (-4.2)$ $= 10.5\,\text{N}$ $= 10.5\,\text{N}$ in the positive direction
Use Newton's second law to determine acceleration.	$\vec{a} = \frac{\vec{F}_{\text{net}}}{m}$ $= \frac{10.5}{2.1}$ $= 5.0\,\text{m s}^{-2}$ forwards

Worked example: Try yourself 5.1.5

IDENTIFYING SYSTEMATIC ERROR

The apparatus shown in the figure below was used as an experiment to demonstrate Newton's second law by placing different masses in the trolley and using an accelerometer to measure the acceleration of the trolley.



The data from the experiment are shown in the table below.

Accelerating mass (kg)	Mass of trolley (kg)	Mass in trolley (kg)	Total mass of system (kg)	Acceleration (m s^{-2})
5.0	1.5	0.0	6.5	5.2
5.0	1.5	0.5	7.0	4.9
5.0	1.5	1.0	7.5	4.5
5.0	1.5	1.5	8.0	4.3
5.0	1.5	2.0	8.5	4.0
5.0	1.5	2.5	9.0	3.8

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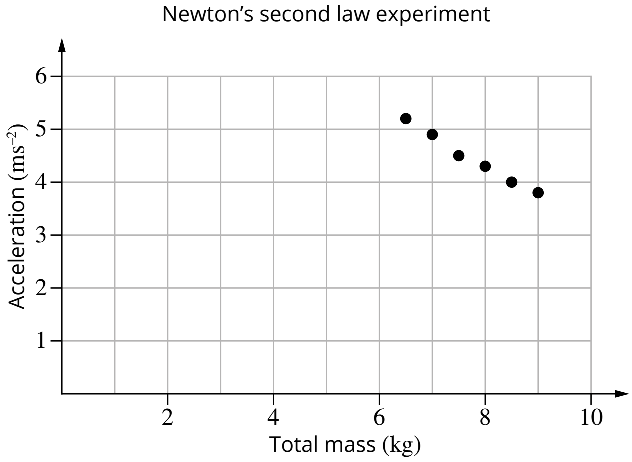
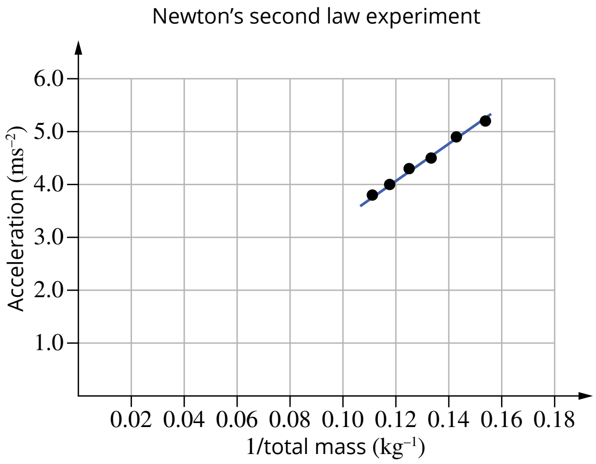
<table>
<tr><td colspan="2">a Construct a graph of the data to show that Newton’s second law applies in this situation.</td></tr>
<tr><td>Thinking</td><td>Working</td></tr>
<tr><td>Recall Newton’s second law.</td><td>$\vec{F}_{net} = m\vec{a}$</td></tr>
<tr><td>Construct a graph of the acceleration versus the total mass of the system.</td><td>
</td></tr>
<tr><td>Inspect the shape of the graph and compare it to a prediction based on Newton’s second law.</td><td>According to Newton’s second law, since the accelerating force in this situation is constant (i.e. the weight of the accelerating mass) then the total mass of the system should be inversely proportional to its acceleration. Therefore, this graph should have the shape of a hyperbola.
This matches the shape of the graph observed.</td></tr>
<tr><td>Work out what you would have to graph to get a straight line. Recall how to find a mathematical model from a linear relationship in Section 1.5 p. 24.</td><td>Newton’s second law predicts that $\vec{F}_{net} = m\vec{a}$.
$\vec{a} = \vec{F}_{net}\frac{1}{m}$
Therefore, $\uparrow \quad \uparrow \quad \uparrow$
$y = m \; x$</td></tr>
<tr><td>Make a new table of the manipulated data.</td><td>
<table>
<tr><th>1/total mass (kg^{-1})</th><th>Acceleration ($m\,s^{-2}$)</th></tr>
<tr><td>0.15</td><td>5.2</td></tr>
<tr><td>0.14</td><td>4.9</td></tr>
<tr><td>0.13</td><td>4.5</td></tr>
<tr><td>0.13</td><td>4.3</td></tr>
<tr><td>0.12</td><td>4.0</td></tr>
<tr><td>0.11</td><td>3.8</td></tr>
</table></td></tr>
<tr><td>Plot the graph of manipulated data.</td><td>
</td></tr>
<tr><td>Interpret the graph.</td><td>Since this graph is a straight line, this shows the acceleration is inversely proportional to the total mass; therefore, it confirms Newton’s second law.</td></tr>
</table>

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b Use the data to calculate the net force acting on the trolley and determine if there is a systematic error in this experiment.	
Thinking	**Working**
Calculate the equation of the line of best fit.	The equation of the line of best fit was calculated using a spreadsheet: $y = 33x$
Find the equation relating $\vec{a}$ and m.	The regression line has the equation $y = 34x$, so the equation relating $\vec{a}$ and m is $\vec{a} = 33\frac{1}{m}$
Relate this to Newton's second law.	According to Newton's second law: $\vec{a} = \vec{F}_{net}\frac{1}{m}$ $\therefore \vec{F}_{net} = 33\,N$
Compare the measured value to the expected value.	The weight of the acceleration mass is given by: $\vec{F}_{net} = \vec{F}_g = m\vec{g}$ $= 5.0 \times 9.8$ $= 49\,N$ Since the measured value is only 33 N, there is a systematic error of: $49 - 33 = 16\,N$
Identify the likely cause of the systematic error.	It is likely that the systematic error in this experiment is due to friction in the wheels of the trolley and between the string and the pulley.

5.1 Review

1 **a** decrease
b decrease
c increase
d increase

2 **a** kinetic
b kinetic
c static

3 Without friction the car would follow Newton's first law and travel in a straight line at a constant speed. This would cause it to slide across and off the road.

4 incorrect calibration of instruments
zeroing error
friction
air resistance
repeated errors in experimental method, e.g. repeatedly failing to adjust for parallax error.

5 Constant speed, so $\vec{F}_{net} = 0$, then frictional force = 20 N and the applied force = –20 N.

6 **a** $\vec{F}_{net} = \vec{F}_g = m\vec{g}$
Take the direction of motion as the positive direction.
$= 0.50 \times 9.8$
$= 4.9\,N$
$\vec{a} = \frac{\vec{F}_{net}}{m} = \frac{4.9}{(2.5 + 0.5)}$
$= 1.6\,m\,s^{-2}$ in the positive direction

b $\vec{F}_{net} = m\vec{a} = \vec{F}_g$ – friction
so $\vec{F}_{net} = 4.9 - 4.3 = 0.6\,N$
$\vec{a} = \frac{\vec{F}_{net}}{m}$
$= \frac{0.6}{3}$
$= 0.2\,m\,s^{-2}$ in the positive direction

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5.2 Work

Worked example: Try yourself 5.2.1

CALCULATING WORK

A person pushes a heavy wardrobe from one room to another by applying a force of 50 N for a distance of 5 m. Calculate the amount of work done by the person.

Thinking	Working
Recall the definition of work.	$W = \vec{F}_{net}\vec{s}$
Substitute in the values for this situation.	$W = 50 \times 5$
Solve the problem, giving an answer with appropriate units.	$W = 250\,\text{J}$

5.2 Review

1 Energy is the capacity to cause change whereas work occurs when there is an energy change, e.g. energy changes from one form to another or is transferred from one object to another.

2 **a** kinetic (i.e. rotational kinetic)
b potential (i.e. gravitational potential)
c kinetic (i.e. thermal energy in the motion of water particles)
d kinetic (i.e. kinetic energy of air particles)
e potential (i.e. chemical)

3 $W = \vec{F}_{\text{net}}\,\vec{s}$
$= 500 \times 20$
$= 10\,000\,\text{J}$

4 The person exerts a force on the wall but the wall has no displacement ($\vec{s} = 0$), so no work is done.

5 $W = \vec{F}\,\vec{s}$

$\vec{F} = \frac{W}{\vec{s}}$

$= \frac{2700}{150}$

$= 18\,\text{N}$ in the direction of motion

6 Since the box does not move, no work is done.

7 $\vec{F}_{net} = \vec{F}_g = m\vec{g} = 50 \times 9.8 = 490\,\text{N}$

$\vec{s} = \frac{W}{\vec{F}_{\text{net}}}$

$= \frac{735}{490}$

$= 1.5\,\text{m}$

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5.3 Energy changes

Worked example: Try yourself 5.3.1

CALCULATING KINETIC ENERGY

An 80 kg person is crossing the street, walking at 5.0 km h^{-1}. Calculate the person's kinetic energy, giving your answer correct to two significant figures.

Thinking	Working
Convert the person's speed to m s^{-1}.	5.0 km h^{-1} = $\frac{5.0\text{ km}}{1\text{ h}}$ = $\frac{5000\text{ m}}{3600\text{ s}}$ = 1.4 m s^{-1}
Recall the equation for kinetic energy.	$K = \frac{1}{2}m\vec{v}^2$
Substitute the values for this situation into the equation.	$K = \frac{1}{2} \times 80 \times 1.4^2$
State the answer with appropriate units.	K = 78 J

Worked example: Try yourself 5.3.2

CALCULATING KINETIC ENERGY CHANGES

As a bus with a mass of 10 tonnes approaches a school, it slows from 60 km h^{-1} to 40 km h^{-1}.

a Use the work–energy theorem to calculate the work done by the brakes of the bus. Give your answer to two significant figures.	
Thinking	**Working**
Convert the values into SI units.	u = 60 km h^{-1} = $\frac{60\text{km}}{1\text{h}}$ = $\frac{60\,000\text{ m}}{3600\text{ s}}$ = 16.67 m s^{-1} v = 40 kmh^{-1} = $\frac{40\text{km}}{1\text{h}}$ = $\frac{40\,000\text{ m}}{3600\text{ s}}$ = 11.11 m s^{-1} m = 10 tonnes = 10 000 kg
Recall the work–energy theorem.	$W = \frac{1}{2}m\vec{v}^2 - \frac{1}{2}m\vec{u}^2$
Substitute the values for this situation into the equation.	$W = \frac{1}{2}(10\,000 \times 11.11^2) - \frac{1}{2}(10\,000 \times 16.67^2)$
State the answer with appropriate units.	W = –771605 = –770 kJ

b The bus travels 40 m as it decelerates. Calculate the average force applied by the truck's brakes.	
Thinking	**Working**
Recall the definition of work.	$W = \vec{F}_{\text{net}}\,\vec{s}$
Substitute the values for this situation into the equation. Note: Enter all values as positives.	770 kJ = $\vec{F}_{net} \times 40$
Transpose the equation to find the answer.	$\vec{F}_{net} = \frac{770\,000}{40}$ = 19 kN

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Worked example: Try yourself 5.3.3

CALCULATING SPEED FROM KINETIC ENERGY

A 300 kg motorbike has 150 kJ of kinetic energy.
Calculate the speed of the motorbike in km h^{-1}. Give your answer correct to two significant figures.

Thinking	Working
Recall the equation for kinetic energy.	$K = \frac{1}{2}mv^2$
Transpose the equation to make *v* the subject.	$v = \sqrt{\frac{2K}{m}}$
Substitute the values for this situation into the equation.	$v = \sqrt{\frac{2 \times 150000}{300}} = 31.6\,\text{m s}^{-1}$
State the answer in the required units.	$v = 31.6 \times 3.6 = 114\,\text{km h}^{-1}$

Worked example: Try yourself 5.3.4

CALCULATING GRAVITATIONAL POTENTIAL ENERGY

A grocery shelf-stacker lifts a 5 kg bag of dog food onto a shelf 30 cm above the floor.
Calculate the gravitational potential energy of the bag when it is on the shelf. Give your answer correct to two significant figures.

Thinking	Working
Recall the formula for gravitational potential energy.	$\Delta U = m\vec{g}\Delta\vec{h}$
Identify the relevant values for this situation. Only the mass of the bag of dog food is being lifted. Take the floor as the zero potential energy level.	$m = 5\,\text{kg}$ $\vec{g} = 9.8\,\text{N kg}^{-1}$ $\Delta\vec{h} = 30\,\text{cm} = 0.3\,\text{m}$
Substitute the values for this situation into the equation.	$\Delta U = 5 \times 9.8 \times 0.3$
State the answer with appropriate units and significant figures.	$\Delta U = 15\,\text{J}$

Worked example: Try yourself 5.3.5

CALCULATING GRAVITATIONAL POTENTIAL ENERGY RELATIVE TO A REFERENCE LEVEL

A father picks up his baby from its bed. The baby has a mass of 6.0 kg and the mattress of the bed is 70 cm above the ground. When the father holds the baby in his arms, it is 125 cm off the ground.
Calculate the increase in gravitational potential energy of the baby.
Use $g = 9.8\,\text{N kg}^{-1}$ and give your answer correct to two significant figures.

Thinking	Working
Recall the formula for gravitational potential energy.	$\Delta U = mg\Delta h$
Identify the relevant values for this situation. Subtract the baby's original position off the ground from its final position.	$m = 6\,\text{kg}$ $g = 9.8\,\text{N kg}^{-1}$ $\Delta h = 55\,\text{cm} = 0.55\,\text{m}$
Substitute the values for this situation into the equation.	$\Delta U = 6 \times 9.8 \times 0.55$
State the answer with appropriate units and significant figures.	$\Delta U = 32\,\text{J}$

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5.3 Review

1 $80\,\text{km}\,\text{h}^{-1} = 22.22\,\text{m}\,\text{s}^{-1}$

$K = \frac{1}{2}m\vec{v}^2$

$= \frac{1}{2}\ 230 \times 22.22^2 = 56\,779\,\text{J}$ or $57\,\text{kJ}$

2 $W = \frac{1}{2}m\vec{v}^2 - \frac{1}{2}m\vec{u}^2$

$= \frac{1}{2} \times 1500 \times 28^2 - \frac{1}{2}1500 \times 17^2$

$= 371\,250 = 370\,\text{kJ}$

3 $v = \sqrt{\frac{2K}{m}}$

$= \sqrt{\frac{2 \times 5000}{(72 + 9)}}$

$= 11\,\text{m}\,\text{s}^{-1} = 40\,\text{km}\,\text{h}^{-1}$

4 $K = \frac{1}{2}m\vec{v}^2$

$\therefore K \propto m$

Assuming the velocity stays the same, doubling the mass causes K to increase by a factor of 2 as well.

5 **a** $\Delta U = mg\Delta h$

$= 0.057 \times 9.8 \times 8.2 = 4.6\,\text{J}$

b $\Delta U = mg\Delta h$

$= 0.057 \times 9.8 \times 4.1 = 2.3\,\text{J}$

6 $\Delta U = mg\Delta h$

$= 65 \times 9.8 \times (8848 - 5150)$

$= 2.36 \times 10^6\,\text{J}$

$= 2360\,\text{kJ}$

5.4 Mechanical energy and power

Worked example: Try yourself 5.4.1

MECHANICAL ENERGY OF A FALLING OBJECT

A 6.8 kg bowling ball is dropped from a height of 0.75 m. Calculate the kinetic energy of the bowling ball at the instant it hits the ground.

Thinking	Working
Since the ball is dropped, its initial kinetic energy is zero.	$K_{\text{initial}} = 0\,\text{J}$
Calculate the initial gravitational potential energy of the ball.	$U_{\text{initial}} = mgh$ $= 6.8 \times 9.8 \times 0.75$ $= 50\,\text{J}$
Calculate the initial mechanical energy.	$(E_{\text{m}})_{\text{initial}} = K_{\text{initial}} + U_{\text{initial}}$ $= 0 + 50$ $= 50\,\text{J}$
At the instant the ball hits the ground, its gravitational potential energy is zero.	$U_{\text{final}} = 0\,\text{J}$
Mechanical energy is conserved in this situation.	$\therefore (E_{\text{m}})_{\text{initial}} = (E_{\text{m}})_{\text{final}} = 50$ $= K_{\text{final}} + 0$ $K_{\text{final}} = 50\,\text{J}$

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Worked example: Try yourself 5.4.2

FINAL VELOCITY OF A FALLING OBJECT

A 6.8 kg bowling ball is dropped from a height of 0.75 m. Calculate the speed of the bowling ball at the instant before it hits the ground.

Thinking	Working
Recall the formula for the velocity of a falling object.	$v = \sqrt{2gh}$
Substitute the relevant values into the formula and solve.	$v = \sqrt{2 \times 9.8 \times 0.75}$ $= 3.8\,\text{m s}^{-1}$
Interpret the answer.	The bowling ball will be falling at $3.8\,\text{m s}^{-1}$ just before it hits the ground.

Worked example: Try yourself 5.4.3

USING MECHANICAL ENERGY TO ANALYSE PROJECTILE MOTION

An arrow with a mass of 35 g is fired into the air at $80\,\text{m s}^{-1}$ from a height of 1.4 m. Calculate the speed of the arrow when it has reached a height of 30 m.

Thinking	Working
Recall the formula for mechanical energy.	$E_m = K + U = \frac{1}{2}mv^2 + mgh$
Substitute in the values for the arrow as it is fired.	$(E_m)_{\text{initial}} = K_{\text{initial}} + U_{\text{initial}}$ $= \frac{1}{2}mv^2 + mgh$ $= \frac{1}{2}(0.035 \times 80^2) + (0.035 \times 9.8 \times 1.4)$ $= 111.5\,\text{J}$
Use the law of conservation of mechanical energy to find an equation for the final speed.	$(E_m)_{\text{initial}} = (E_m)_{\text{final}}$ $= \frac{1}{2}mv^2 + mgh$ $\frac{1}{2}mv^2 = (E_m)_{\text{initial}} - mgh$ $v^2 = \frac{2\,[(E_m)_{\text{initial}} - mgh]}{m}$
Solve the equation algebraically to find the final speed.	$v^2 = \frac{2\,(111.5 - 0.035 \times 9.8 \times 30)}{0.035}$ $= 5.78 \times 10^3$ $v = 76\,\text{m s}^{-1}$

Worked example: Try yourself 5.4.4

CALCULATING POWER

A weightlifter lifts a 50 kg barbell from the floor to a height of 2.0 m above the ground in 1.4 s. Calculate the power required for this lift. (Assume $g = 9.8\,\text{m s}^{-2}$.)

Thinking	Working
Calculate the force applied.	$\vec{F}_g = m\vec{g}$ $= 50 \times 9.8$ $= 490\,\text{N}$
Calculate the work done.	$W = \vec{F}_{\text{net}}\,\vec{s}$ $= 490 \times 2.0$ $= 980\,\text{J}$

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Recall the formula for power.	$P = \frac{\Delta E}{t}$
Substitute the appropriate values into the formula.	$P = \frac{980}{1.4}$
Solve.	$P = 700\,\text{W}$

Worked example: Try yourself 5.4.5

UNIFORM ACCELERATION AND POWER

A car with a mass of 2080 kg accelerates uniformly from 0 to 100 km h^{-1} in 3.7 s.
What is the power exerted by the engine of the car? Ignore rolling resistance and air resistance, and state your answer correct to three significant figures.

Thinking	Working
Convert the car's final speed to m s^{-1}.	$100\,\text{km h}^{-1} = \frac{100\,\text{km}}{1\,\text{h}} = \frac{100\,000\,\text{m}}{3600\,\text{s}}$ $= 27.778\,\text{m s}^{-1}$
Calculate the change in kinetic energy of the car.	$K = \frac{1}{2}mv^2 - \frac{1}{2}mu^2$ $= \frac{1}{2} \times 2080 \times 27.778^2 - \frac{1}{2} \times 2080 \times 0$ $= 802.5\,\text{kJ}$
Substitute the appropriate values into the power formula.	$P = \frac{\Delta E}{t}$ $= \frac{802.5 \times 10^3}{3.7}$
Solve.	$P = 217\,\text{kW}$

Worked example: Try yourself 5.4.6

FORCE–VELOCITY FORMULATION OF POWER

A heavy wardrobe is pushed along the floor at an average speed of 2.2 m s^{-1}. The force of kinetic friction is 1200 N.
What amount of power is required for this task? Give your answer correct to two significant figures.

Thinking	Working
Recall the force–velocity formulation of the power equation.	$P = \vec{F}\vec{v}$
Substitute the appropriate values into the formula.	$P = 1200 \times 2.2$
Solve.	$P = 2600\,\text{W}$

Worked example: Try yourself 5.4.7

FORCE–VELOCITY FORMULATION OF POWER WITH AIR RESISTANCE/ROLLING RESISTANCE

A car with a mass of 900 kg is travelling at a constant speed of 15 m s^{-1}. Rolling resistance and air resistance combine to oppose the motion of the car with a force of 750 N.
What is the power output of the car's engine at this speed?

Thinking	Working
Remember that for the car to be travelling at constant speed, the car must be in equilibrium.	$\sum \vec{F} = \vec{F}_{\text{forwards}} + \vec{F}_{\text{drag}} = 0$ $\vec{F}_{\text{forwards}} = -\vec{F}_{\text{drag}}$
Recall the force–velocity formulation of the power equation.	$P = \vec{F}\vec{v}$
Substitute the appropriate values into the formula.	$P = 750 \times 15$
Solve.	$P = 11\,250\,\text{W} = 11\,\text{kW}$

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5.4 Review

1 **a** $K_{final} = U_{initial} = mgh$

$= 180 \times 9.8 \times 15 = 26\,460\,J$

To two significant figures: 26 000 J

b $U_{initial} = K_{final}$

$26\,460 = K_{final} + mgh$

$= K_{final} + (180 \times 9.8 \times 5)$

$26\,460 - 8820 = K_{final}$

$K_{final} = 17\,640\,J$

2 $v = \sqrt{2gh}$

$h = \frac{v^2}{2g}$

$= \frac{5.4^2}{2 \times 9.8}$

$= 1.5\,m$

3 **a** $E_m = K + U$

$= \frac{1}{2}mv^2 + mgh$

$= \frac{1}{2} \times 0.800 \times 28.5^2 + 0.800 \times 9.8 \times 1.45$

$= 336\,J$

b $E_m = K + U$

$= \frac{1}{2}mv^2 + mgh$

$336 = \frac{1}{2} \times 0.800 \times v^2 + 0$

$v = \sqrt{\frac{336}{0.400}}$

$= 29.0\,m\,s^{-1}$

4 $100\,km\,h^{-1} \div 3.6 = 27.8\,m\,s^{-1}$

$P = \frac{\Delta E}{t}$

$= \frac{\frac{1}{2}(1610 \times 27.8^2)}{5.50}$

$= 113\,kW$

5 $F_g = mg = 500 \times 9.8 = 4900\,N$

$P = \vec{F}\,\vec{v} = 4900 \times 5$

$= 24\,500\,W = 24.5\,kW$

6 $80\,km\,h^{-1} \div 3.6 = 22.2\,m\,s^{-1}$

Since the car is maintaining a constant speed, the force produced by the car's engine must be equal to the combined force of air resistance and rolling resistance.

$P = \vec{F}\vec{v}$

$\therefore \vec{F} = \frac{P}{v}$

$= \frac{40\,000}{22}$

$= 1800\,N$ against the direction of motion

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CHAPTER 5 REVIEW

1 The billycart is moving at a constant speed, so $\vec{F}_{net} = 0$, then frictional force = applied force = 25 N in the direction of motion.

2 Friction between the car's tyres and the road is the unbalanced force which is causing the car to decelerate. If there is almost no friction, the car will not slow down; it will continue to travel in a straight line at a constant speed.

3 $\vec{a} = \frac{\vec{F}_{net}}{m}$

$= \frac{150}{100}$

$\vec{a} = 1.5\,\text{m s}^{-2}$ (Only the magnitude of the acceleration is required, so it does not need a direction.)

4 $\vec{F}_{net} = 150 - 45$

$= 105\,\text{N}$

$\vec{a} = \frac{\vec{F}_{net}}{m}$

$= \frac{105}{100}$

$\vec{a} = 1.05\,\text{m s}^{-2}$ (Only the magnitude of the acceleration is required, so it does not need a direction.)

5 Using Newton's second law:

$\vec{F}_{net} = m\vec{a}$

$= 100 \times 0.6$

$= 60.0\,\text{N}$

Net force is also given by the sum of individual forces:

$\vec{F}_{net} = 150 - \text{friction}$

$\text{friction} = 150 - \vec{F}_{net}$

$= 150 - 60$

$= 90\,\text{N}$ (Only the magnitude of the force is required, so it does not need a direction.)

6 Using Newton's second law:

$\vec{F}_{net} = m\vec{a}$

$= 125 \times 0.800$

$= +100\,\text{N}$

Net force is also given by the sum of individual forces:

$\vec{F}_{net} = \vec{F}_{forwards} - 30$

$\vec{F}_{forwards} = \vec{F}_{net} + 30$

$= 100 + 30$

$\vec{F}_{forwards} = +130\,\text{N}$

7 $W = \vec{F}_{net}\,\vec{s} = 2000 \times 80 = 160\,000\,\text{J}$ or $160\,\text{kJ}$

8 $W = \vec{F}_{net}\,\vec{s}$

$= m\vec{g} \times \vec{s}$

$= 200 \times 9.8 \times 30$

$= 58\,800\,\text{J}$ or $58.8\,\text{kJ}$

9 For each step: $W = \vec{F}_{net}\vec{s} = 60 \times 9.8 \times 0.165 = 97\,\text{J}$

For all 12 steps: $W = 12 \times 97 = 1164\,\text{J}$ or $1.16\,\text{kJ}$

10 $\vec{F}_g = m\vec{g} = 50 \times 9.8 = 490\,\text{N}$

$W = \vec{F}_{net}\,\vec{s}$

$\vec{s} = \frac{W}{\vec{F}_{net}}$

$= \frac{4000}{490}$

$= 8.2\,\text{m}$

11 $150\,\text{km h}^{-1} = 41.67\,\text{m s}^{-1}$

$156\,\text{g} = 0.156\,\text{kg}$

$K = \frac{1}{2}mv^2$

$= \frac{1}{2} \times 0.156 \times 41.67^2 = 135\,\text{J}$

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12 $v = \sqrt{\frac{2K}{m}} = \sqrt{\frac{2 \times 70\,000}{1200}} = 11\,\mathrm{m\,s^{-1}}$

13 $K = \frac{1}{2}mv^2$

$\therefore K \propto v^2$

So doubling the velocity causes the kinetic energy to change by a factor of 2^2 or 4.

14 $\Delta U = m\vec{g}\Delta\vec{h} = 88 \times 9.8 \times 0.40 = 340\,\mathrm{J}$

15 $\Delta U = m\vec{g}\Delta\vec{h}$

$\therefore \Delta h = \frac{\Delta U}{mg} = \frac{100}{2.0 \times 9.8}$

$= 5.1\,\mathrm{m}$

16 $v = \sqrt{2gh} = \sqrt{2 \times 9.8 \times 0.76} = 3.9\,\mathrm{m\,s^{-1}}$

17 $v = \sqrt{2gh}$

$\therefore h = \frac{v^2}{2g} = \frac{9.0^2}{2 \times 9.8}$

$= 4.1\,\mathrm{m}$

18 $E_\mathrm{m} = K + U$

$= \frac{1}{2}mv^2 + mgh$

$= \frac{1}{2}(0\,.43 \times 16^2) + (0.43 \times 9.8 \times 0)$

$= 55\,\mathrm{J}$

$E_\mathrm{m} = K + U$

$55 = \frac{1}{2}\,(0.43 \times v^2) + (0.43 \times 9.8 \times 2.44)$

$= 0.215v^2 + 10.3$

$v = \sqrt{\frac{44.7}{0.215}}$

$= 14.4\,\mathrm{m\,s^{-1}}$

19 **a** $\Delta U = mg\Delta h = (75 + 0.005) \times 9.8 \times 0.11$

$= 81\,\mathrm{J}$

b The gain in gravitational potential energy of the pendulum (81 J) is equal to the kinetic energy of the pendulum as it starts to swing upwards so the pendulum had 81 J of energy.

c The kinetic energy of the bullet just before it hit the pendulum was 81 J.

$K = \frac{1}{2}mv^2$

$\therefore v = \sqrt{\frac{2K}{m}}$

$v = \sqrt{\frac{2 \times 81}{0.005}}$

$v = 180\,\mathrm{m\,s^{-1}}$

20 Remember: $P = \frac{\Delta E}{t} = \frac{mgh}{t}$ and $1\,\mathrm{kW} = 1000\,\mathrm{W}$

$P = \frac{mgh}{t}$

$= \frac{5000 \times 9.8 \times 20}{5}$

$= 196\,000\,\mathrm{W}$

$= 196\,\mathrm{kW}$

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21 $100\ \text{km h}^{-1} = \frac{100}{3.6}\ \text{m s}^{-1}$

$= 27.8\ \text{m s}^{-1}$

$\Delta K = \frac{1}{2}mv^2 - \frac{1}{2}mu^2$

$= \frac{1}{2}(720 \times 27.8^2) - 0$

$= 278\ \text{kJ}$

$P = \frac{\Delta E}{t}$

$= \frac{278\,000}{18.0}$

$= 15.4\ \text{kW}$

22 $\vec{F} = \frac{P}{\vec{v}}$

$\vec{F} = \frac{25000}{17}$

$= 1500\ \text{N}$ against the direction of motion

23 **a** $W = \Delta K = \frac{1}{2}mv^2 = \frac{1}{2} \times 60 \times 8^2 = 1920\ \text{J}$

b $\vec{F}_{\text{net}} = \frac{W}{\vec{s}}$

$= \frac{1920}{20}$

$= 96\ \text{N}$ in the direction of motion

24 $\Delta U = m\vec{g}\Delta\vec{h} = 120 \times 1.6 \times 0.1 = 19.2\ \text{J}$

25 **a** $v = \sqrt{2gh} = \sqrt{2 \times 9.8 \times 15} = 17\ \text{m s}^{-1}$

b $v = \sqrt{2gh} = \sqrt{2 \times 9.8 \times 10} = 14\ \text{m s}^{-1}$

26 $\vec{F}_N = -\vec{F}_g = 5 \times 9.8 = 49\ \text{N}$ upwards

The speed is constant, so the force needed is equal and opposite to the friction force.

friction $= \mu\vec{F}_N = 0.5 \times 49 = 24.5\ \text{N}$

$P = \vec{F}\vec{v}$

$= 24.5 \times 3 = 73.5\ \text{W}$

27 Responses will vary.

When a force is applied to the hovercraft over a distance, work is being done on the hovercraft, increasing its energy. The larger the distance, the larger the energy transfer, according to the equation $W = \vec{F}\vec{s}$.

The kinetic energy of your push is transferred directly to the hovercraft. Some energy will be lost to air resistance, but frictional forces between the surface and the hovercraft are almost zero because the balloon is not in physical contact with the surface.

The distance the hovercraft travels can be predicted by analysing the net force acting on the balloon hovercraft, or by using the law of conservation of mechanical energy.

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Chapter 6 Momentum, energy and simple systems

6.1 Conservation of momentum

Worked example: Try yourself 6.1.1

CALCULATING MOMENTUM

Calculate the momentum of a 1230 kg car travelling north 16.7 m s^{-1}.

Thinking	Working
Ensure that the variables are in their standard units.	m = 1230 kg $\vec{v}$ = 16.7 m s^{-1} north
Apply the equation for momentum.	$\vec{p} = m\vec{v}$ $= 1230 \times 16.7$ $= 20\,541$ $= 20\,500$ kg m s^{-1} north

Worked example: Try yourself 6.1.2

CONSERVATION OF MOMENTUM

A 1200 kg wrecking ball moving north at 2.50 m s^{-1} collides with a 1500 kg wrecking ball moving south at 4.00 m s^{-1}. Calculate the velocity of the 1500 kg ball after the collision, if the 1200 kg ball rebounds at 3.50 m s^{-1} south.

Thinking	Working
Identify the variables using subscripts. Ensure that the variables are in their standard units.	m_1 = 1200 kg $\vec{u}_1$ = 2.50 m s^{-1} north $\vec{v}_1$ = 3.50 m s^{-1} south m_2 = 1500 kg $\vec{u}_2$ = 4.00 m s^{-1} south $\vec{v}_2$ = ?
Apply the sign convention to the variables.	m_1 = 1200 kg $\vec{u}_1$ = +2.50 m s^{-1} $\vec{v}_1$ = –3.50 m s^{-1} m_2 = 1500 kg $\vec{u}_2$ = –4.00 m s^{-1} $\vec{v}_3$ = ?
Apply the equation for conservation of momentum.	$\sum \vec{p}_{before} = \sum \vec{p}_{after}$ $m_1\vec{u}_1 + m_2\vec{u}_2 = m_1\vec{v}_1 + m_2\vec{v}_2$ $(1200 \times 2.50) + (1500 \times -4.00) = (1200 \times -3.50) + 1500\vec{v}_2$ $1500\vec{v}_3 = 3000 + -6000 - (-4200)$ $\vec{v}_2 = \frac{1200}{1500}$ $= +0.800$ m s^{-1}
Apply the sign convention to describe the direction of the final velocity.	$\vec{v}_2$ = 0.80 m s^{-1} north

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Worked example: Try yourself 6.1.3

CONSERVATION OF MOMENTUM WHEN MASSES COMBINE

A 90.0 kg rugby player running north at 1.50 m s^{-1} tackles an opponent with a mass of 80.0 kg who is running south at 5.00 m s^{-1}. The players are locked together after the tackle.
Calculate the velocity of the players immediately after the tackle.

Thinking	Working
Identify the variables using subscripts and ensure that the variables are in their standard units. Add m_1 and m_2 to get m_{1+2}.	$m_1 = 90.0\,\text{kg}$ $\vec{u}_1 = 1.50\,\text{m s}^{-1}$ north $m_2 = 80.0\,\text{kg}$ $\vec{u}_2 = 5.00\,\text{m s}^{-1}$ south $m_{1+2} = 170\,\text{kg}$ $\vec{v} = ?$
Apply the sign convention to the variables.	$m_1 = 90.0\,\text{kg}$ $\vec{u}_1 = +1.50\,\text{m s}^{-1}$ $m_2 = 80.0\,\text{kg}$ $\vec{u}_2 = -5.00\,\text{m s}^{-1}$ $m_{1+2} = 170\,\text{kg}$ $\vec{v} = ?$
Apply the equation for conservation of momentum.	$\sum \vec{p}_{\text{before}} = \sum \vec{p}_{\text{after}}$ $m_1\vec{u}_1 + m_2\vec{u}_2 = m_{1+2}\vec{v}$ $(90 \times 1.50) + (80 \times -5.00) = 170\vec{v}$ $\vec{v} = \frac{135 + (-400)}{170}$ $= -1.56\,\text{m s}^{-1}$
Apply the sign to describe the direction of the final velocity.	$\vec{v}_3 = 1.56\,\text{m s}^{-1}$ south

Worked example: Try yourself 6.1.4

CONSERVATION OF MOMENTUM FOR EXPLOSIVE COLLISIONS

A stationary 2000 kg cannon fires a 10.0 kg cannonball. After firing, the cannon recoils north at 8.15 m s^{-1}.
Calculate the velocity of the cannonball just after it is fired.

Thinking	Working
Identify the variables using subscripts and ensure that the variables are in their standard units. Note that m_1 is the sum of the masses of the cannon and the cannonball.	$m_1 = 2010\,\text{kg}$ $\vec{u}_1 = 0\,\text{m s}^{-1}$ $m_2 = 2000\,\text{kg}$ $\vec{v}_2 = 8.15\,\text{m s}^{-1}$ north $m_3 = 10\,\text{kg}$ $\vec{v} = ?$
Apply the sign convention to the variables.	$m_1 = 2010\,\text{kg}$ $\vec{u}_1 = 0\,\text{m s}^{-1}$ $m_2 = 2000\,\text{kg}$ $\vec{v}_2 = +8.15\,\text{m s}^{-1}$ $m_3 = 10\,\text{kg}$ $\vec{v}_3 = ?$

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Apply the equation for conservation of momentum for explosive collisions.	$\sum \vec{p}_{before} = \sum \vec{p}_{after}$ $m_1\vec{u}_1 = m_2\vec{v}_2 + m_3\vec{v}_3$ $2010 \times 0 = (2000 \times 8.15) + 10\vec{v}_3$ $\vec{v}_3 = \frac{0-16300}{10}$ $= \frac{-16\,300}{10}$ $= -1630\,\text{m s}^{-1}$
Apply the sign to describe the direction of the final velocity.	$\vec{v}_3 = 1630\,\text{m s}^{-1}$ south

Worked example: Try yourself 6.1.5

CONSERVATION OF KINETIC ENERGY

An object with a mass of 2200 kg was travelling at 17.56 m s^{-1} when it hit a stationary object with a mass of 2150 kg. The two objects joined together and moved away at 8.881 m s^{-1} north. Was this collision elastic, or was it inelastic?

Thinking	Working
Identify the variables using subscripts. Ensure that the variables are in their standard units.	$m_1 = 2200\,\text{kg}$ $\vec{u}_1 = 17.56\,\text{m s}^{-1}$ north $\vec{v}_1 = 8.881\,\text{m s}^{-1}$ north $m_2 = 2150\,\text{kg}$ $\vec{u}_2 = 0\,\text{m s}^{-1}$ south $\vec{v}_2 = 8.881\,\text{m s}^{-1}$ north
Apply the sign convention to the variables.	$m_1 = 2200\,\text{kg}$ $\vec{u}_1 = +2.2\,\text{m s}^{-1}$ $\vec{v}_1 = +1.549\,\text{m s}^{-1}$ $m_2 = 0.5\,\text{kg}$ $\vec{u}_2 = -3.0\,\text{m s}^{-1}$ $\vec{v}_2 = +6.75\,\text{m s}^{-1}$
Apply the equation for conservation of kinetic energy.	$\sum \frac{1}{2}m\vec{v}^2_{\text{before}} = \sum \frac{1}{2}m\vec{v}^2_{\text{after}}$ $\sum \frac{1}{2}m\vec{v}^2_{\text{before}} = \frac{1}{2}m_1\vec{u}_1^2 + \frac{1}{2}m_2\vec{u}_2^2$ $= \frac{1}{2}\times 2200 \times 17.56^2 + \frac{1}{2}\times 2150 \times 0.0^2$ $= 339188.96 + 0$ $= 339\,188\,\text{J}$ $\sum \frac{1}{2}m\vec{v}^2_{\text{after}} = \frac{1}{2}m_1\vec{v}_1^2 + \frac{1}{2}m_2\vec{v}_2^2$ $= \frac{1}{2}\times 4350 \times 8.881^2$ $= 171\,547\,\text{J}$
Determine whether the collision was elastic or inelastic.	The total kinetic energy after the collision was less than the total kinetic energy before, so kinetic energy was lost during the collision. The collision was therefore inelastic.

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6.1 Review

1 $\vec{p} = m\vec{v}$

$= 3.50 \times 2.50$

$= 8.75\,\text{kg m s}^{-1}$ south

2 $\vec{p} = m\vec{v}$

$= 433 \times 22.2$

$= 9612.6$

$= 9610\,\text{kg m s}^{-1}$ west

3 $\vec{p} = m\vec{v}$

$= 0.058 \times 61.0$

$= 3.54\,\text{kg m s}^{-1}$ south

4 First ball: $\vec{p} = m\vec{v} = 4.5 \times 3.5 = 16\,\text{kg m s}^{-1}$

Second ball: $\vec{p} = m\vec{v} = 2.5 \times 6.8 = 17\,\text{kg m s}^{-1}$

The second ball has the greater momentum. The direction for each momentum will be in same the direction as the velocity, which has not been supplied.

5 $m_1\vec{u}_1 = m_2\vec{v}_2 + m_3\vec{v}_3$

$(470.0 \times 0) = (70.0 \times 2.50) + 400\vec{v}_3$

$400\vec{v}_3 = -175$

$\vec{v}_3 = \frac{-175}{400}$

$= -0.438\,\text{m s}^{-1}$

The boat moves backwards at $0.438\,\text{m s}^{-1}$.

6 $m_1\vec{u}_1 + m_2\vec{u}_2 = m_1\vec{v}_1 + m_2\vec{v}_2$

$(0.250 \times 0) + (1.50 \times 1.20) = (0.250 \times 1.60) + 1.50\vec{v}_2$

$1.50\vec{v}_3 = 0 + 1.80 - 0.40$

$\vec{v}_2 = \frac{1.40}{1.50}$

$= 0.93\,\text{m s}^{-1}$, north

7 $\sum \vec{p}_{\text{before}} = \sum \vec{p}_{\text{after}}$

$m_1\vec{u}_1 + m_2\vec{u}_2 = m_3\vec{v}_3$

$(25\,000 \times 2.00) + (m_2 \times 0) = (25000 + m_2) \times 0.300$

$50\,000 = 0.300 \times 25\,000 + 0.300m_2$

$0.300m_2 = 50\,000 - 7500$

$m_2 = \frac{42500}{0.300}$

$= 141\,667$

$= 142\,000\,\text{kg}$ (to three significant figures)

8 $\sum \vec{p}_{\text{before}} = \sum \vec{p}_{\text{after}}$

$m_1\vec{u}_1 = m_2\vec{v}_2 + m_3\vec{v}_3$

$0 = (9995)\vec{v}_2 + (5.0)(6000)$

$\vec{v}_2$ = velocity of space shuttle = $3.0\,\text{m s}^{-1}$ in the direction opposite to that of the exhaust gases.

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6.2 Change in momentum

Worked example: Try yourself 6.2.1

IMPULSE OR CHANGE IN MOMENTUM

A student with a mass of 55.0 kg hurries to class after lunch, moving at 4.55 m s^{-1} north. Suddenly she remembers that she has forgotten her laptop, and runs back to her locker at 6.15 m s^{-1} south.

Calculate the impulse of the student during the time it takes her to turn around.

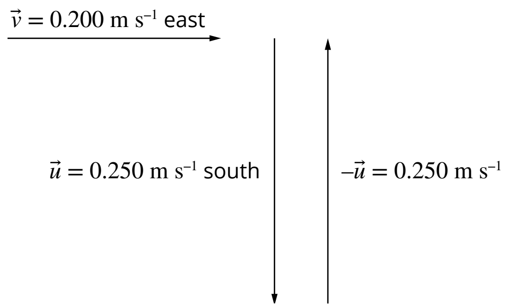
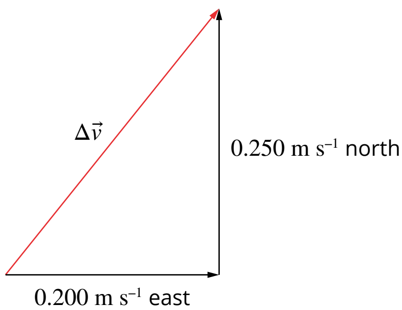
Thinking	Working
Ensure that the variables are in their standard units.	$m = 55.0$ kg $\vec{u} = 4.55$ m s^{-1} north $\vec{v} = 6.15$ m s^{-1} south
Apply the sign convention to the velocity vectors.	$m = 55.0$ kg $\vec{u} = 4.55$ m s^{-1} $\vec{v} = -6.15$ m s^{-1}
Apply the equation for impulse or change in momentum.	$\Delta\vec{p} = m\vec{v} - m\vec{u}$ $= (55.0 \times -6.15) - (55.0 \times 4.55)$ $= -338.25 - 250.25$ $= -588.5$ kg m s^{-1} $= -589$ kg m s^{-1}
Apply the sign convention to describe the direction of the impulse.	impulse = 589 kg m s^{-1} south

Worked example: Try yourself 6.2.2

IMPULSE OR CHANGE IN MOMENTUM IN TWO DIMENSIONS

A 160 g pool ball rolling south at 0.250 m s^{-1} bounces off a cushion and rolls east at 0.200 m s^{-1}.

Calculate the impulse on the ball during its impact with the cushion.

Thinking	Working
Identify the formula for calculating a change in velocity $\Delta\vec{v}$.	$\Delta\vec{v}$ = final velocity – initial velocity
Draw the final velocity vector $\vec{v}$ and the initial velocity vector $\vec{u}$ separately. Then draw the initial velocity in the opposite direction, which represents the negative of the initial velocity $\vec{u}$.	
Construct a vector diagram, drawing $\vec{v}$ first and then from its head draw the opposite of $\vec{u}$. The change of velocity vector is drawn from the tail of the final velocity to the head of the opposite of the initial velocity.	

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Because the two vectors to be added are at 90° to each other, apply Pythagoras' theorem to calculate the magnitude of the change in velocity.	$\Delta\vec{v}^2 = 0.2^2 + 0.25^2$ $= 0.0400 + 0.0625$ $\Delta\vec{v} = \sqrt{0.1025}$ $= 0.320\,\text{m}\,\text{s}^{-1}$
Calculate the angle from the north vector to the change in velocity vector.	$\tan\theta = \frac{0.200}{0.250}$ $\theta = \tan^{-1}0.800$ $= 38.7°$
State the magnitude and direction of the change in velocity.	$\Delta\vec{v} = 0.320\,\text{m}\,\text{s}^{-1}$ N38.7°E
Identify the variables using subscripts and ensure that the variables are in their standard units.	$m_1 = 0.160\,\text{kg}$ $\Delta\vec{v} = 0.320\,\text{m}\,\text{s}^{-1}$ N38.7°E
Apply the equation for impulse or change in momentum.	$\Delta\vec{p} = m\vec{v} - m\vec{u}$ $= m(\vec{v} - \vec{u})$ $= m\Delta\vec{v}$ $= 0.160 \times 0.320$ $= 0.0512\,\text{kg}\,\text{m}\,\text{s}^{-1}$
Apply the direction convention to describe the direction of the change in momentum.	$\Delta\vec{p} = 0.0512\,\text{kg}\,\text{m}\,\text{s}^{-1}$ N38.7°E

6.2 Review

1 $\Delta\vec{p} = m\vec{v} - m\vec{u}$

$= (9.50 \times -6.25) - (9.50 \times 2.50)$

$= -59.375 - 23.75$

$= -83.1\,\text{kg}\,\text{m}\,\text{s}^{-1}$

$= 83.1\,\text{kg}\,\text{m}\,\text{s}^{-1}$ south

2 $\Delta\vec{p} = m\vec{v} - m\vec{u}$

$= (6050 \times 16.7) - (6050 \times -22.2)$

$= 101\,035 + 134\,310 = 235\,345$

$= 235\,000\,\text{kg}\,\text{m}\,\text{s}^{-1}$ east

3 $\Delta\vec{p} = m\vec{v} - m\vec{u}$

$= (8.00 \times 8.00) - (8.00 \times 3.00)$

$= 64.0 - 24.0$

$= 40.0\,\text{kg}\,\text{m}\,\text{s}^{-1}$ east

4 $\Delta\vec{p} = m\vec{v} - m\vec{u}$

$= (0.250 \times -9.80) - (0.250 \times 0)$

$= -2.45\,\text{kg}\,\text{m}\,\text{s}^{-1}$

$= 2.45\,\text{kg}\,\text{m}\,\text{s}^{-1}$ down

5 $\Delta\vec{p} = m\vec{v} - m\vec{u}$

$m\vec{v} = \Delta\vec{p} + m\vec{u}$

$\vec{v} = \frac{\Delta\vec{p} + m\vec{u}}{m}$

$= \frac{-0.075 + 0.125 \times 3.00}{0.125}$

$= 2.4\,\text{m}\,\text{s}^{-1}$ north

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6 $\Delta\vec{v}$ = final velocity – initial velocity

$= \vec{v} - \vec{u}$

$= \vec{v} + (-\vec{u})$

$= 45.0\,\text{m s}^{-1}$ north + $45.0\,\text{m s}^{-1}$ east

The magnitude of the change in velocity is calculated using Pythagoras' theorem:

$\Delta\vec{v}^2 = 45.0^2 + 45.0^2$

$= 2025 + 2025$

$\Delta\vec{v} = \sqrt{4050}$

$= 63.6\,\text{m s}^{-1}$

Use trigonometry to calculate the angle of the change in momentum.

$\tan\theta = \frac{45.0}{45.0}$

$\theta = \tan^{-1} 1$

$= 45°$

$\Delta\vec{v} = 63.6\,\text{m s}^{-1}$ N45°E

The magnitude of the change in momentum is calculated using:

$\Delta\vec{p} = m\vec{v} - m\vec{u}$

$= m(\vec{v} - \vec{u})$

$= m\Delta\vec{v}$

$= 45.0 \times 63.6$

$= 2863.8$

$\Delta\vec{p} = 2860\,\text{kg m s}^{-1}$ N45°E

7 $\Delta\vec{v}$ = final velocity – initial velocity

$= \vec{v} - \vec{u}$

$= \vec{v} + (-\vec{u})$

$= 3.60\,\text{m s}^{-1}$ west + $4.00\,\text{m s}^{-1}$ south

The magnitude of the change in velocity is calculated using Pythagoras' theorem:

$\Delta\vec{v}^2 = 3.60^2 + 4.00^2$

$= 12.96 + 16.0$

$\Delta\vec{v} = \sqrt{28.96}$

$= 5.38\,\text{m s}^{-1}$

Use trigonometry to calculate the angle of the change in momentum.

$\tan\theta = \frac{3.60}{4.00}$

$\theta = \tan^{-1} 0.9$

$= 42°$

$\Delta\vec{v} = 5.38\,\text{m s}^{-1}$ S42°W

The magnitude of the change in momentum is calculated using:

$\Delta\vec{p} = m\vec{v} - m\vec{u}$

$= m(\vec{v} - \vec{u})$

$= m\Delta\vec{v}$

$= 70.0 \times 5.38$

$= 377\,\text{kg m s}^{-1}$

$\Delta\vec{p} = 377\,\text{kg m s}^{-1}$ S42°W

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6.3 Momentum and net force

Worked example: Try yourself 6.3.1

CALCULATING THE FORCE AND IMPULSE

A student drops a 56.0 g egg onto a table from a height of 60 cm. Just before it hits the table, the velocity of the egg is 3.43 m s^{-1} down. The egg's final velocity is zero, which it reaches in 3.55 milliseconds.

a Calculate the impulse of the egg.	
Thinking	**Working**
Ensure that the variables are in their standard units.	$m = 0.0560$ kg $\vec{u} = 3.43$ m s^{-1} downwards $\vec{v} = 0$ m s^{-1}
Apply the sign and direction convention for motion in one dimension. Up is positive and down is negative.	$m = 0.0560$ kg $\vec{u} = -3.43$ m s^{-1} $\vec{v} = 0$ m s^{-1}
Apply the equation for change in momentum.	$\Delta\vec{p} = m(\vec{v} - \vec{u})$ $= 0.0560 \times (0 - (-3.43))$ $= 0.192$ kg m s^{-1}
Refer to the sign and direction convention to determine the direction of the change in momentum. This is equal to the impulse.	impulse = $\Delta\vec{p} = 0.192$ kg m s^{-1} upwards

b Calculate the average force that acts to cause the impulse.	
Thinking	**Working**
Use the answer to part **a**. Ensure that the variables are in their standard units.	$\Delta\vec{p} = 0.192$ kg m s^{-1} $\Delta t = 3.55 \times 10^{-3}$ s
Apply the equation for force.	$\vec{F}\Delta t = \Delta\vec{p}$ $\vec{F} = \frac{\Delta\vec{p}}{\Delta t}$ $= \frac{0.192}{3.55 \times 10^{-3}}$ $= +54.1$ N
Refer to the sign and direction convention to determine the direction of the force.	$\vec{F} = 54.1$ N upwards

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Worked example: Try yourself 6.3.2

CALCULATING THE FORCE AND IMPULSE (SOFT LANDING)

A student drops a 56.0 g egg into a mound of flour from a height of 60 cm. Just before it hits the mound of flour, the velocity of the egg is 3.43 m s^{-1} down. The egg comes to rest in the flour in 0.325 seconds.

a Calculate the impulse of the egg.	
Thinking	**Working**
Ensure that the variables are in their standard units.	$m = 0.0560$ kg $\vec{u} = 3.43$ m s^{-1} downwards $\vec{v} = 0$ m s^{-1}
Apply the sign and direction convention for motion in one dimension. Up is positive and down is negative.	$m = 0.0560$ kg $\vec{u} = -3.43$ m s^{-1} $\vec{v} = 0$ m s^{-1}
Apply the equation for change in momentum.	$\Delta\vec{p} = m(\vec{v} - \vec{u})$ $= 0.0560 \times [0 - (-3.43)]$ $= 0.192$ kg m s^{-1}
Refer to the sign and direction convention to determine the direction of the change in momentum. This is equal to the impulse.	impulse = $\Delta\vec{p} = 0.192$ kg m s^{-1} upwards

b Calculate the average force that acts to cause the impulse.	
Thinking	**Working**
Using the answer to part **a**, ensure that the variables are in their standard units.	$\Delta\vec{p} = 0.192$ kg m s^{-1} $\Delta t = 0.325$ s
Apply the equation for force.	$\vec{F}\Delta t = \Delta\vec{p}$ $\vec{F} = \frac{\Delta\vec{p}}{\Delta t}$ $= \frac{0.192}{0.325}$ $= +0.591$ N
Refer to the sign and direction convention to determine the direction of the force.	$\vec{F} = 0.591$ N upwards

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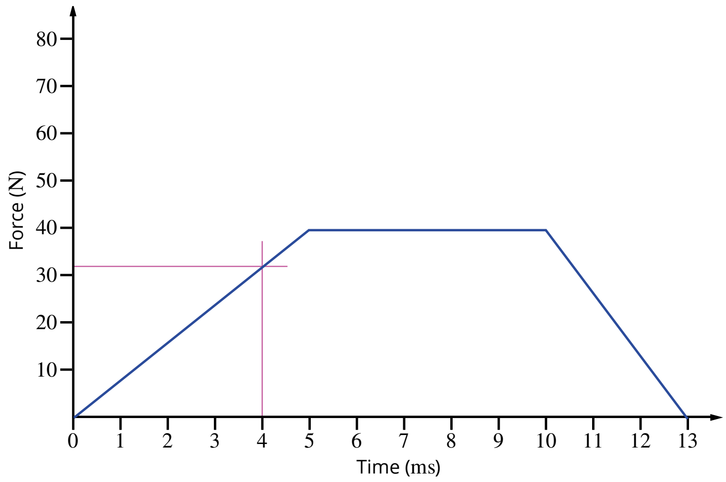
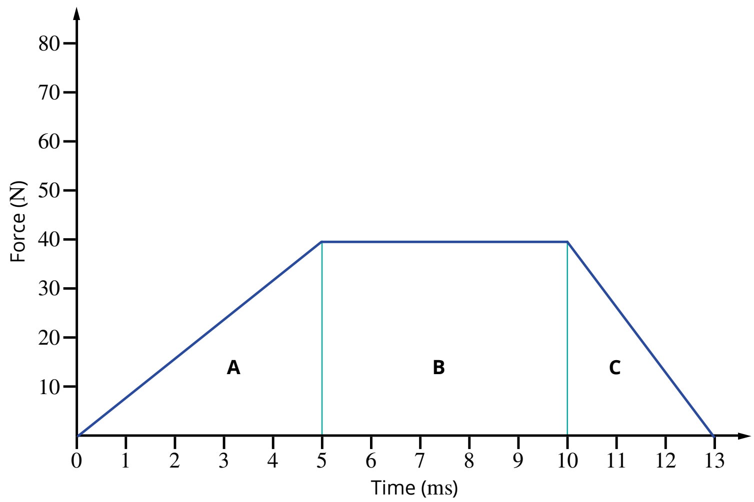
Worked example: Try yourself 6.3.3

CALCULATING THE TOTAL IMPULSE FROM A CHANGING FORCE

A student records the force acting on a tennis ball as it bounces off a hard concrete floor over a period of time. The graph shows the forces acting on a ball during its collision with the concrete floor.

<table>
<tr><td colspan="2">a Determine the force acting on the ball at a time of 4.0 milliseconds.</td></tr>
<tr><td>Thinking</td><td>Working</td></tr>
<tr><td>From the 4.0 millisecond point on the x-axis go up to the line of the graph, then across to the y-axis.</td><td>
</td></tr>
<tr><td>The force is estimated by reading the intercept of the y-axis</td><td>$\vec{F} = +32\,\text{N}$</td></tr>
</table>

<table>
<tr><td colspan="2">b Calculate the total impulse of the ball over the 13 millisecond period of time.</td></tr>
<tr><td>Thinking</td><td>Working</td></tr>
<tr><td>Break the area under the graph into sections for which you can calculate the area: A, B and C.</td><td>
</td></tr>
<tr><td>Calculate the area of the three sections A, B and C using the equations for the area of a triangle and the area of a rectangle.</td><td>Area = A + B + C
$= \left(\frac{1}{2}b\times h\right)+(b\times h)+\left(\frac{1}{2}b\times h\right)$
$= \left[0.5\times(5\times10^{-3})\times 40\right]+\left[(5\times10^{-3})\times 40\right]+\left[0.5\times(3\times10^{-3})\times 40\right]$
$= 0.1 + 0.2 + 0.06$
$= 0.36$</td></tr>
<tr><td>The total impulse is equal to the area.</td><td>impulse = area
$= +0.36\,\text{kg m s}^{-1}$</td></tr>
<tr><td>Apply the sign and direction convention for motion in one dimension vertically.</td><td>impulse = $0.36\,\text{kg m s}^{-1}$ upwards</td></tr>
</table>

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6.3 Review

1 **a** $\Delta\vec{p} = m(\vec{v} - \vec{u})$
$= 45.0 \times (12.5 - 2.45) = 452.25$
$= 452\,\text{kg}\,\text{m}\,\text{s}^{-1}$ east

b impulse $= \Delta\vec{p}$
$= 452\,\text{kg}\,\text{m}\,\text{s}^{-1}$ east

c $\vec{F}\Delta t$ = impulse
$\vec{F} = \frac{\Delta\vec{p}}{\Delta t}$
$= \frac{450}{3.50}$
$= 129\,\text{N}$ east

2 Airbags are designed to increase the duration of the collision, which reduces the rate of change of momentum of a person's head during a car accident. Increasing the duration of the collision decreases the force, which reduces the severity of injury.

3 **a** $\Delta\vec{p} = m(\vec{v} - \vec{u})$
$= 0.156 \times (0 - (-12.2))$
$= 1.90\,\text{kg}\,\text{m}\,\text{s}^{-1}$ east

b impulse $= \Delta\vec{p}$
$= 1.90\,\text{kg}\,\text{m}\,\text{s}^{-1}$ east

c $\vec{F}\Delta t$ = impulse
$\vec{F} = \frac{\Delta\vec{p}}{\Delta t}$
$= \frac{1.90}{0.100}$
$= 19.0\,\text{N}$ east

d $\vec{F}_{av}\Delta t$ = impulse
$\vec{F}_{av} = \frac{\Delta\vec{p}}{\Delta t}$
$= \frac{1.90}{0.300}$
$= 6.34\,\text{N}$ east

4 **a** Impulse $= \vec{F}_{av}\Delta t = \Delta\vec{p}$
$= 0.13 \times 25$
$= +3.3\,\text{kg m}^{-1}$

b $\vec{F} = \frac{3.3}{0.05}$
$= 65\,\text{N}$ in the direction of the ball's travel

c 65 N in the opposite direction to the ball's travel.

5 Results for question 5 are estimates and may vary.

a Maximum force = 1200 N

b Impulse $= \vec{F}\Delta t$ = area under force–time graph = 63 N s

6 **a** $\Delta\vec{p} = m(\vec{v} - \vec{u})$
$= (0.025)(0 - 50)$
$= 1.25\,\text{kg}\,\text{m}\,\text{s}^{-1}$ opposite in direction to its initial velocity

b Impulse $= \vec{F}\Delta t = \Delta\vec{p}$
$= 1.25\,\text{kg}\,\text{m}\,\text{s}^{-1}$ opposite in direction to its initial velocity

c $\vec{v}^2 = \vec{u}^2 + 2\vec{a}\vec{s}$
$0 = 50^2 + 2\vec{a}(2.0 \times 10^{-2})$
$\vec{a} = -6.25 \times 10^4\,\text{m}\,\text{s}^{-2}$
$\vec{F} = m\vec{a}$
$= 0.025(-6.25 \times 10^4)$
$= 1.6 \times 10^3\,\text{N}$ in the opposite direction to the initial velocity of the arrow.

This could also be solved by instead finding the time it takes for the arrow to come to a stop.

7 **a** The crash helmet is designed so that the stopping time is increased by the collapsing shell during impact. This will reduce the force, since impulse $= \vec{F}\Delta t = \Delta\vec{p}$.

b No. A rigid shell would reduce the stopping time, therefore increasing the force.

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CHAPTER 6 REVIEW

1 $\vec{v} = 11.11\,\text{m s}^{-1}$

$\vec{p} = m\vec{v}$

$= 6000 \times 11.11$

$= 66666\,\text{kg m s}^{-1}$

$= 6.7 \times 10^4\,\text{kg m s}^{-1}$

2 $m = 2.1 \times 10^{-5}$ kg

$\vec{p} = m\vec{v}$

$= 2.1 \times 10^{-5} \times 1.8$

$= 3.8 \times 10^{-5}\ \text{kg m s}^{-1}$

3 Chestnut horse:

$\vec{p} = m\vec{v}$

$= 800 \times 20$

$= 1.6 \times 10^3\ \text{kg m s}^{-1}$

Grey horse:

$\vec{p} = m\vec{v}$

$= 600 \times 25$

$= 1.5 \times 10^3\,\text{kg m s}^{-1}$

The chestnut horse has the greater momentum.

4 $\Delta\vec{p} = m(\vec{v} - \vec{u})$

$= 155 \times (3.25 - 6.50)$

$= 504\,\text{kg m s}^{-1}$ west

5 $\Delta\vec{p} = m(\vec{v} - \vec{u})$

$= (25.5 \times -2.25) - (25.5 \times 6.40)$

$= -221\,\text{kg m s}^{-1}$

$= 221\,\text{kg m s}^{-1}$ backwards

6 $\sum \vec{p}_{before} = \sum \vec{p}_{after}$

$(40 + 5 + 2) \times 5 = (40 + 5) \times \vec{v} + 2 \times 0$

$235 = 45\vec{v}$

$\vec{v} = 5.2\ \text{ms}^{-1}$

7 $\sum \vec{p}_{before} = \sum \vec{p}_{after}$

$m_1\vec{u}_1 + m_2\vec{u}_2 = m_1\vec{v}_1 + m_2\vec{v}_2$

$(40.0 \times 0) + (154 \times 0) = (40.0 \times 2.15) + 154\vec{v}_2$

$154\vec{v}_2 = 0 + -86$

$\vec{v}_2 = \frac{-86}{154}$

$= -0.558\,\text{m s}^{-1}$

The astronaut moves backwards at $0.558\,\text{m s}^{-1}$.

8 **a** The rocket loses 50 kg over the 2 s period, so the average mass of the rocket/fuel in this time can be used in this calculation. The average mass is 225 kg.

$\sum \vec{p}_{before} = \sum \vec{p}_{after}$

$m_1\vec{u}_1 = m_2\vec{v}_2 + m_3\vec{v}_3$

$0 = 225\vec{v}_2 + (50)(-180)$

$\vec{v}_2 = +40\,\text{m s}^{-1}$

b $\vec{F}_{net} = m\vec{a} = m \times \frac{(\vec{v}-\vec{u})}{t}$

$= 225 \times \left(\frac{40}{2}\right)$

$= +4.5 \times 10^3\,\text{N}$

c Net upwards acceleration $= \frac{\text{resultant force}}{\text{mass}}$

$= \frac{4.5 \times 10^3 - (225 \times 9.81)}{225}$

$= 10.2\,\text{m s}^{-2}$

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9 $\sum \vec{p}_{before} = \sum \vec{p}_{after}$

$m_1\vec{u}_1 + m_2\vec{u}_2 = m_1\vec{v}_1 + m_2\vec{v}_2$

$7 \times 2 + 0.5 \times 0 = 7\vec{v}_1 + 0.5 \times 5$

$14 = 7\vec{v}_1 + 2.5$

$\vec{v}_1 = 1.6\,\text{m}\,\text{s}^{-1}$ north

10 $\Delta\vec{p} = m\vec{v} - m\vec{u}$

$= (0.06 \times -30) - (0.06 \times 50)$

$= -1.8 - 3$

$= -4.8\,\text{kg}\,\text{m}\,\text{s}^{-1}$

$\Delta\vec{p} = 4.8\,\text{kg}\,\text{m}\,\text{s}^{-1}$ away from racquet

11 $\Delta\vec{v}$ = final velocity – initial velocity

$= \vec{v} - \vec{u}$

$= \vec{v} + (-\vec{u})$

$= 5.00\,\text{m}\,\text{s}^{-1}$ north $+ 4.00\,\text{m}\,\text{s}^{-1}$ east

The magnitude of the change in velocity is calculated using Pythagoras' theorem:

$\Delta\vec{v}^2 = 5.00^2 + 4.00^2$

$= 25.0 + 16.0$

$\Delta\vec{v} = \sqrt{41.0}$

$= 6.40\,\text{m}\,\text{s}^{-1}$

Use trigonometry to calculate the angle of the change in momentum.

$\tan\theta = \frac{4.00}{5.00}$

$\theta = \tan^{-1} 0.9$

$= 38.7°$

$\Delta\vec{v} = 6.40\,\text{m}\,\text{s}^{-1}$ N38.7°E

The magnitude of the change in momentum is calculated using:

$\Delta\vec{p} = m\vec{v} - m\vec{u}$

$= m(\vec{v} - \vec{u})$

$= m\Delta\vec{v}$

$= 75.0 \times 6.40$

$= 480\,\text{kg}\,\text{m}\,\text{s}^{-1}$

$= 480\,\text{kg}\,\text{m}\,\text{s}^{-1}$ N38.7°E

12 $\Delta\vec{v}$ = final velocity – initial velocity

$= \vec{v} - \vec{u}$

$= \vec{v} + (-\vec{u})$

$= 8.5\,\text{m}\,\text{s}^{-1}$ east $+ 6.0\,\text{m}\,\text{s}^{-1}$ south

The magnitude of the change in velocity is calculated using Pythagoras' theorem:

$\Delta\vec{v}^2 = 8.5^2 + 6.0^2$

$= 75.25 + 36$

$\Delta\vec{v} = \sqrt{108.25}$

$= 10.4\,\text{m}\,\text{s}^{-1}$

Use trigonometry to calculate the angle of the change in momentum.

$\tan\theta = \frac{8.5}{6.0}$

$\theta = \tan^{-1} 1.4$

$= 54.8°$

$\Delta\vec{v} = 10.4\,\text{m}\,\text{s}^{-1}$ S54.8°W

$\Delta\vec{p} = m\vec{v} - m\vec{u}$

$= m(\vec{v} - \vec{u})$

$= m\Delta\vec{v}$

$= 50.0 \times 10.4$

$= 520.2\,\text{kg}\,\text{m}\,\text{s}^{-1}$

$= 520\,\text{kg}\,\text{m}\,\text{s}^{-1}$ S55°W

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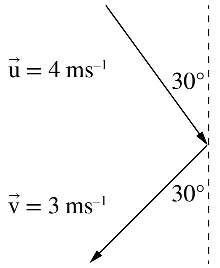
13



From the diagram, choose directions for positive and negative. This solution uses down and left as positive directions.

Break the initial velocity into its components

$\vec{u}_y = 4\cos(30) = 3.46$

$\vec{u}_x = -4\sin(30) = -2.0$

Break the final velocity into its components

$\vec{v}_y = 3\cos(30) = 2.6$

$\vec{v}_x = 3\sin(30) = 1.5$

$\Delta\vec{v}_y = 2.6 - 3.46$

$= -0.86$

$\Delta\vec{v}_x = 1.5 - (-2.0)$

$= 3.5$

$\Delta\vec{v}^2 = (-0.86)^2 + 3.5^2$

$\Delta\vec{v} = 3.6\,\mathrm{m\,s^{-1}}$

$\tan\theta = \frac{3.5}{0.86}$

$\theta = \tan^{-1} 4.07$

$= 76°$ from the cushion

$\Delta\vec{p} = m\Delta\vec{v}$

$= 0.16 \times 3.6$

$= 0.58\,\mathrm{kg\,m\,s^{-1}}$, 76° from the cushion

14 $\Delta\vec{p} = m(\vec{v} - \vec{u})$

$= 0.270 \times (0 - (-5.60))$

$= 1.51\,\mathrm{kg\,m\,s^{-1}}$ east

$\vec{F}_{av}\Delta t = \Delta\vec{p}$

$\vec{F}_{av} = \frac{\Delta\vec{p}}{\Delta t}$

$= \frac{1.51}{1.00}$

$= 1.51\,\mathrm{N}$ east

15 impulse $= \Delta\vec{p}$

= area under force–time graph

$= 0.5 \times 0.04 \times 500$

$= 10\,\mathrm{kg\,m\,s^{-1}}$

16 As the bat and ball form an isolated system, momentum is conserved. The gain in momentum of the ball is equal to the loss of momentum of the bat. Hence:

$\Delta\vec{p} = 10\,\mathrm{kg\,m\,s^{-1}}$

17 $\Delta\vec{p} = m\Delta\vec{v}$

$\therefore \Delta\vec{v} = \frac{\Delta\vec{p}}{m}$

$= \frac{10}{0.170}$

$= 58.8$

$= 59\,\mathrm{m\,s^{-1}}$

18 **a** $\sum \vec{p}_{\text{before}} = \sum \vec{p}_{\text{after}}$

$m_1\vec{u}_1 + m_2\vec{u}_2 = m_1\vec{v}_1 + m_2\vec{v}_2$

$80 \times 6 + 70 \times 3 = 80 \times 4 + 70\vec{v}_2$

$\vec{v}_2 = 5.3\,\mathrm{m\,s^{-1}}$

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b The sum of kinetic energy before the collision equals the sum of the kinetic energy after when the collision is 100% elastic.

$$\sum \tfrac{1}{2} m\vec{v}^2_{\text{before}} = \tfrac{1}{2} m_1 \vec{u}_1{}^2 + \tfrac{1}{2} m_2 \vec{u}_2{}^2$$
$$= \tfrac{1}{2} \times 80 \times 6^2 + \tfrac{1}{2} \times 70 \times 3^2$$
$$= 1755\,\text{J}$$
$$\sum \tfrac{1}{2} m\vec{v}^2_{\text{after}} = \tfrac{1}{2} m_1 \vec{v}_1^2 + \tfrac{1}{2} m_2 \vec{v}_2^2$$
$$= \tfrac{1}{2} \times 80 \times 3^2 + \tfrac{1}{2} \times 70 \times 5.3^2$$
$$= 1618\,\text{J}$$

There is a loss of energy in the collision, so it is an inelastic collision.

19 **a** $\Delta\vec{v} = \vec{v} - \vec{u}$

$= 1 - (-5)5$

$= 6\,\text{m}\,\text{s}^{-1}$ upwards

impulse $= m\Delta\vec{v}$

$= 5 \times 6$

$= 30\,\text{kg}\,\text{m}\,\text{s}^{-1}$ upwards

b impulse $= \vec{F}\Delta t$

$30 = \vec{F} \times 0.1$

$\vec{F} = 300\,\text{N}$ upwards

20 To stop the egg an impulse is experienced:

impulse $= \Delta\vec{p} = m\Delta\vec{v}$

$= 0.05 \times 5$

$= 0.25\,\text{kg}\,\text{m}\,\text{s}^{-1}$

Increasing the time of the collision, reduces the force on the egg according to the equation impulse $= \vec{F}\Delta t$. To ensure the force on the egg is less than 2 N, the time of the collision must be greater than:

$0.25 = 2 \times \Delta t$

$\Delta t = 0.125\,\text{s}$

Cushioning the fall of the egg will increase the time taken for the collision to occur. Examples of material are bubble-wrap, cotton wool, pillow, pile of sawdust or flour.

21 Responses will vary.

The system in the activity is of the combined mass of the student and the medicine ball. The momentum of an object depends on the mass and the velocity. The heavier the mass, the larger the momentum. Objects with a larger momentum require larger momentum transfer to come to a stop. This momentum transfer required a force to be applied for a period of time, the larger the momentum transfer the larger the force or the longer the time. In this inquiry students will be able to feel the increased difficulty in stopping, and by measuring the stopping distance they will have a method of quantifying that feeling.

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Module 2 Review answers

Dynamics

MULTIPLE CHOICE

1 C. Newton's third law states: 'For every action force (object A on B) there is an equal and opposite reaction force (object B on A)'. If boat A exerts a force of $+\vec{F}$ N on boat B, then boat B exerts an equal force $(-\vec{F})$ back on boat A. The negative sign indicates that the force acts in the opposite direction.

2 **a** D. Newton's second law states that $\sum \vec{F} = m\vec{a}$, or an object experiences no acceleration when the net force on the object is zero.

For a person in a lift, if they are not accelerating (e.g. are at rest or travelling at constant velocity) then the net force on them must be zero. When this occurs, the magnitude of the weight force $m\vec{g}$ is equal to the magnitude of the normal force, so the scales read 60 kg.

b A. Newton's second law states that $\sum \vec{F} = m\vec{a}$.

The two forces acting on the student are $m\vec{g}$ (downwards) and normal force (upwards)

$\therefore \vec{N} - m\vec{g} = m\vec{a}$

or $\vec{N} = m\vec{g} + m\vec{a}$

3 D. In orbit at a constant radius and constant speed, both the gravitational potential energy and the kinetic energy of the moon remain unchanged. Because Earth's gravitational force on the Moon acts at right angles to the velocity of the Moon, it does no work on the Moon.

($W = \vec{F}\,\vec{s}\cos\theta$, and $\theta = 90°$, so work done = 0.)

4 A. Momentum is a vector, so direction of momentum must be considered.

Taking direction to the right as positive:

Total momentum = $(+m\vec{v}) + (-m\vec{v}) = 0\,\text{kg m s}^{-1}$

5 C. Hooke's law states that $\vec{F} = k\vec{x}$.

$k = 10\,\text{N m}^{-1}$

$\vec{x} = +20\,\text{cm} = +0.2\,\text{m}$

$\vec{F} = 10 \times 0.2$

$= +2\,\text{N}$

$= m\vec{g}$

$2 = m \times 9.8$

$m = 0.20\,\text{kg}$

$= 200\,\text{g}$

6 A, B and C. (A magnet moving iron filings is an example of a force mediated by a field.)

7 A, C and D. B is the weight and normal force. These are not a reaction pair. D shows that the weight force is actually a pair with the gravitational force an object exerts on the Earth.

8 A, B and C. Constant velocity means the net force is equal to zero.

9 D. Velocity is a vector, so if it is constant then both its magnitude and its direction must remain the same.

10 C.

$\vec{F}_{net} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3$

$= 20\,\text{N left} + 15\,\text{N right} + 10\,\text{N up}$

$= (20 - 15)\text{ left} + 10\text{ up}$

$= 5\text{ left} + 10\text{ up}$

$F^2 = 5^2 + 10^2$

$F = \sqrt{125}$

$= 11.2\,\text{N}$

$\tan\theta = \frac{10}{5}$

$\theta = \tan^{-1} 2.0$

$= 63.43°$

$90 - 63.43 = \text{N}26.6°\text{W}$

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11 C. $W = \vec{F}\,\vec{s}$

$= 1800 \times 1000$

$= 1\,800\,000$

$= 1.8\,\text{MJ}$

12 D. This is the only one in which there is no displacement, so the work done is zero.

13 A. $W = \vec{F}\,\vec{s}\sin\theta$

$= 17.5 \times 8 \times \sin 35$

$= 80.3$

14 B. This diagram shows that a constant force lifted the box to a displacement of 1.4 m.

15 C.

$60 \div 3.6 = 16.67\,\text{m s}^{-1}$

$K = \frac{1}{2}m\vec{v}^2$

$= \frac{1}{2} \times 1000 \times 16.67^2$

$= 138\,889$

$= 139\,\text{kJ}$

16 D. $30 \div 3.6 = 8.33\,\text{m s}^{-1}$

$p = mv$

$= 3000 \times 8.33$

$= 25000\,\text{kg m s}^{-1}$ (the magnitude is a scalar quantity so it does not require a direction)

17 C. According to the law of conservation of momentum, the total momentum of the two bodies before the collision must be equal to the momentum of the combined body after the collision.

18 B.

impulse $= m\vec{v} - m\vec{u}$

$= (3.8 \times -4.9) - (3.8 \times 2.8)$

$= 29.3\,\text{kg m s}^{-1}$ north

19 A.

$m_1\vec{u}_1 + m_2\vec{u}_2 = m_3\vec{v}_3$

$120 \times 2.1 + 100 \times\ -3.5 = 220 \times \vec{v}_3$

$252 - 350 = 220\vec{v}_3$

$\vec{v}_3 = -0.45$

$= 0.5\,\text{m s}^{-1}$ west

20 C.

$m_1\vec{u}_1 = m_2\vec{u}_2 + m_3\vec{v}_3$

$(80 + 1) \times 7.6 = 80 \times 7.1 + 1 \times \vec{v}_3$

$\vec{v}_3 = 615.6 - 568$

$= 47.6\,\text{m s}^{-1}$ north

SHORT ANSWER

21 **a** $\vec{F}_{EM}$ is the gravitational force by the Earth on the man, and $\vec{F}_N = \vec{F}_{SM}$, the normal reaction force exerted by the surface on the man.

b $\vec{F}_{EM} = 980\,\text{N} = -\vec{F}_N$ (the forces are equal in magnitude but opposite in direction).

c $\vec{F}_{ME}$ is the gravitational attraction that the man exerts on the Earth, which is the reaction force to his weight; $\vec{F}_{MS}$ is the force that the man exerts on the surface.

22 **a** $\vec{F}_{EA}$ is the gravitational force exerted by the Earth on A, and is directed downwards. (This is equal to the weight force of the block.) This is balanced by an equal normal reaction force $\vec{F}_{BA}$ directed upwards. Both forces are 100 N in magnitude.

b $\vec{F}_{EB}$ is the weight of the block B, which is 100 N downwards. $\vec{F}_{AB}$ is the force exerted by A on B, also 100 N downwards. The normal reaction force $\vec{F}_{CB}$ balances both, so it is 200 N upwards.

c $\vec{F}_{BC}$ is 200 N downwards (effectively caused by the weight of A and B, and is the reaction pair to $\vec{F}_{CB}$), $\vec{F}_{EC} = 100\,\text{N}$ downwards and the normal reaction force $\vec{F}_{TC} = 300\,\text{N}$ is exerted upwards by the table on C.

d The force $\vec{F}_{EB}$ is still exerted on block B, but $\vec{F}_{CB} = 0$. Both A and B fall and so the contact forces between A and B also go to zero. Each block only experiences its own weight force and accelerates under gravity.

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23 **a** Constant velocity means acceleration = 0, so $\sum \vec{F} = 0$.

$\vec{F} = m\vec{g}\sin\theta + \vec{F}_f$

$= (100 \times 9.8 \times \sin 30°) + 110$

$= 600\,\text{N}$

b $\sum \vec{F} = m\vec{a}$

$= 100 \times 2.0$

$= 200 = F - 600 = 800\,\text{N}$

c $\sum \vec{F} = m\vec{a}$

$100\vec{a} = 1000 - 600$

$\vec{a} = \frac{400}{100} = 4.0\,\text{m}\,\text{s}^{-2}$ up the incline

24 **a** Acceleration $= \frac{\text{resultant force}}{\text{total mass}}$

$\therefore \vec{a} = \frac{(20-10)\times 9.8}{30}$

$= 3.3\,\text{m}\,\text{s}^{-2}$ clockwise

i.e. $3.3\,\text{m}\,\text{s}^{-2}$ up for the 10 kg mass and $3.3\,\text{m}\,\text{s}^{-2}$ down for the 20 kg mass.

b The resultant force on the 20 kg mass:

$\vec{F}_{net} = m\vec{g} - \vec{F}_T$

$\vec{F}_T = 20 \times 9.8 - 20 \times 3.3$

$\therefore \vec{F}_T = 1.3 \times 10^2\,\text{N}$

25 **a** $\sum \vec{F}_h = 800\cos 60 - 100 = 300\,\text{N}$ to the right

b $W = 100 \times 10 = 1000\,\text{J} = 1.0\,\text{kJ}$

c $W = 300 \times 10 = 3000\,\text{J} = 3.0\,\text{kJ}$

d $W = \Delta K = 3.0\,\text{kJ}$

e $\Delta K = \frac{1}{2}(mv^2) = 3000\,\text{J}$

$\therefore v = 17.3\,\text{m}\,\text{s}^{-1}$

26 **a** $E_{tot} = K + U = \frac{1}{2}(mv^2) + mgh$

$E_{tot} = \frac{1}{2}(2.0 \times 10^2) + 2.0 \times 9.8 \times 2.0 = 139.2 = 140\,\text{J}$

b The kinetic energy of the sphere at B = kinetic energy of sphere at A – $mg\Delta h$

$= 100 - (2.0 \times 9.8 \times 1.0) = 80\,\text{J}$

c The minimum kinetic energy occurs at C.

Then min $K = 100 - 2.0 \times 9.8 \times 3.0 = 41.2\,\text{J}$

$\therefore$ min speed $= 6.4\,\text{m}\,\text{s}^{-1}$

d As we assume no losses due to friction, E_{tot} remains constant = 140 J.

27 **a** Taking the zero position 0.50 m below the trampoline:

$mg\Delta h = \frac{1}{2}k(\Delta x)^2$

$(34 \times 9.8 \times (3.5 + 0.50) = \frac{1}{2}k(0.50)^2$

$k = 1.1 \times 10^4\,\text{N}\,\text{m}^{-1}$

b Child has maximum kinetic energy just before striking the trampoline on first descent.

c Gravitational potential energy ⇒ kinetic energy ⇒ elastic potential energy of trampoline ⇒ kinetic energy as child rebounds losing contact with trampoline (with some loss to heat and sound) ⇒ gravitational potential energy

28 The swimmer pulls against the water with her arms, exerting a force on the water. The reaction force (Newton's third law) of the water on her arms is what propels her forwards. If this reaction force is greater than the sum of the drag forces on her, she will accelerate according to Newton's second law. If there is no net force she will travel at constant velocity according to Newton's first law.

29 Earth: $F_N = 2.0 \times 10^4 \times 9.8 = 1.96 \times 10^5\,\text{N}$

Moon: $F_N = 2.0 \times 10^4 \times 1.6 = 3.2 \times 10^4\,\text{N}$

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30 **a** Impulse = $\Delta\vec{p} = m\Delta\vec{v} = 86 \times 7.5 = 645\,\text{kg m s}^{-1}$ in the positive direction

b Momentum is always conserved in a collision. The momentum after the collision will be a combination of the momentum of the pole (which causes the pole to wobble) and the momentum of the player as he rebounds off the post.

c Concussion is an injury caused when the brain is rapidly accelerated or decelerated inside the skull. As the player's head comes to rest against the pole, according to Newton's first law his brain would continue in its motion at $7.5\,\text{m s}^{-1}$ until it collides with the inside of the skull, which could result in a concussion.

EXTENDED RESPONSE

31 **a** $K = \frac{1}{2}(mv^2) = \frac{1}{2}(2000 \times 40.0^2) = 1.6\,\text{MJ}$

b Acceleration is found from the gradient of the v–t graph.

$\sum\vec{F} = m\vec{a} = 2000 \times 8.0 = 16000\,\text{N} = 1.6 \times 10^4\,\text{N}$ forwards

c $\sum\vec{F} = 16\,000\,\text{N} = \vec{F} - 400 = 16.4\,\text{kN}$ forwards

d $W = \vec{F}\vec{s} = (1.64 \times 10^4 \times 100) = 1.64 \times 10^6\,\text{J} = 1.64\,\text{MJ}$

This is equal to the change in kinetic energy found in part a.

e $P = t = \frac{1.6 \times 10^6}{5.0} = 320\,\text{kW}$

f $W = 400 \times 100 = 40\,\text{kJ}$

32 **a** The elastic potential energy stored in the spring = area under force–compression graph between $\Delta x = 0$ and $\Delta x = 2.0\,\text{cm}$

Elastic potential, $U = 40\,\text{J}$

b When trolley comes to rest, its initial kinetic energy of 250J will be stored in the spring as elastic potential energy.

c When the trolley comes to rest, $K = 0$. All its initial kinetic energy has been transferred to the spring as elastic potential energy.

Then $U_s = \frac{1}{2}k(\Delta x)^2 = 250\,\text{J}$

Where $k = \frac{\vec{F}}{\vec{x}} = \frac{(10 \times 10^3)}{(5 \times 10^{-2})} = 2 \times 10^5\,\text{N m}^{-1}$

$\therefore\ \Delta x = 0.050\,\text{m} = 5.0\,\text{cm}$

d The elastic potential energy stored in the spring is transferred back to the trolley as kinetic energy when the spring starts to regain its original shape.

e The spring is elastic. This means it can retain its original shape after the compression force has been removed.

33 **a** The section from 2.0 cm to 5.0 cm is rough because the cube loses kinetic energy (K) in this section.

b $K = \frac{1}{2}m\vec{v}^2 = 5\,\text{J}$

$\therefore v^2 = \frac{2K}{m} = \frac{10}{0.2} = 50$, so $v = 7.1\,\text{m s}^{-1}$

c $\Delta K = 5.0 - 2.0 = 3.0\,\text{J}$

d The kinetic energy has been converted into heat and sound.

e $W = \Delta K = \vec{F}\,\vec{x}$

$\therefore \vec{F} = \frac{\Delta K}{\vec{x}} = \frac{3.0}{0.030} = 100\,\text{N}$

34 **a** $\vec{v}^2 = \vec{u}^2 + 2\vec{a} \times \vec{s}$

$(8.0 \times 10^2) = 0^2 + 2\vec{a} \times 20$

$\therefore\ \vec{a} = +1.6 \times 10^4\,\text{m s}^{-2}$

b $\vec{F} = m\vec{a} = 5.5 \times 10^2 \times 1.6 \times 10^4 = +8.8 \times 10^6\,\text{N}$

c $\vec{p} = m\vec{v} = 550 \times 8.0 \times 10^2 = +4.4 \times 10^5\,\text{kg m s}^{-1}$

d Conservation of momentum means that the sum of all momentum in the system will be equal to zero.

Momentum of projectile = –momentum of gun

$1.08 \times 10^5 \times \vec{v} = 4.4 \times 10^5\,\text{kg m s}^{-1}$

$\vec{v} = -4.1\,\text{m s}^{-1}$

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e $\vec{F} = \frac{\Delta\vec{p}}{\Delta t}$

$t = \frac{\vec{v} - \vec{u}}{\vec{a}}$

$= \frac{8.0 \times 10^2}{1.6 \times 10^4} = 0.05\,\text{s}$

$\vec{F} = \frac{4.4 \times 10^5}{0.05}$

$= -8.8 \times 10^6\,\text{N}$ as before

f $W = \vec{F} \times \vec{d}$

$= 8.8 \times 10^6 \times 20 = 1.8 \times 10^8\,\text{J}$

K of projectile $= \frac{1}{2}mv^2 = 1.8 \times 10^8\,\text{J}$

This obviously represents an ideal situation; realistically there would be significant losses.

35 **a** Total momentum before collision = total momentum after collision

$4.0 \times 10^4 \times 3.0 = (4.0 \times 10^4 + 1.5 \times 10^4) \times v_{\text{final}}$

$\therefore v_{\text{final}} = 2.2\,\text{m}\,\text{s}^{-1}$

b K before $= \frac{1}{2}mv^2 = 1.8 \times 10^5\,\text{J}$

K after $= 1.3 \times 10^5\,\text{J}$, so kinetic energy is not conserved.

c Inelastic. Total momentum is always conserved, but kinetic energy is conserved only in elastic collisions. In inelastic collisions some kinetic energy is converted into other forms of energy such as heat and sound.

d $d = v \times t$

$= 2.2 \times 2 \times 60$

$= 262\,\text{m}$

e $\sum \vec{F} = m\vec{a}$

$= (4.0 \times 10^4 + 1.5 \times 10^4) \times -1.5$

$= -8.3 \times 10^4\,\text{N}$

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Chapter 7 Wave properties

7.1 Mechanical waves

7.1 Review

1 The particles oscillate backwards and forwards or upwards and downwards around a central or average position and pass on the energy carried by the wave. They do not move along with the wave.

2 **a** False. Longitudinal waves occur when particles of the medium vibrate in the same direction in which the wave energy is travelling.
  b True. This gives the transverse wave its characteristic shape.
  c True. Sound is a longitudinal wave that travels through air.
  d True. The string vibrates up and down about its normal (unplucked) position.

3 Point B is moving downwards. The crest of the wave is to the right of B, so it has already reached maximum positive displacement, and the trough is to the left of B, so it has yet to reach maximum negative displacement. It must be moving downwards.

4 A has moved right and B has moved left. As the sound wave moves to the right, particles ahead of the compression must move to the left initially to meet the compression and then move forward to carry the compression to the right. Therefore, particle B has moved to the left of its undisturbed position and particle A has now moved to the right of its undisturbed position.

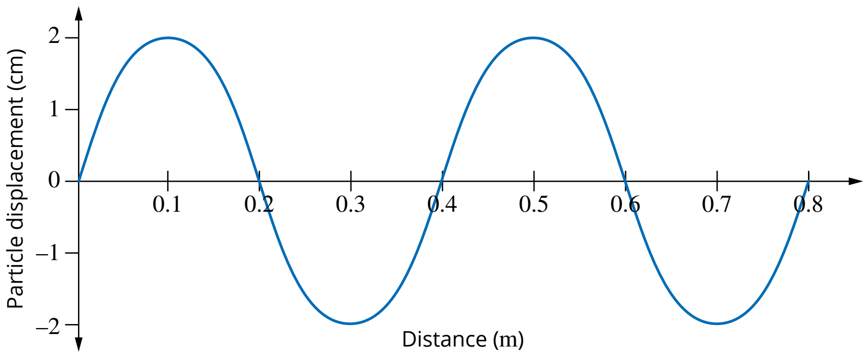
5 C and D. Only energy is transferred by a wave therefore the statements saying that air particles have travelled to Lee are incorrect. Energy has been transferred from the speaker to Lee and it is the air particles that have passed this energy along through the air.

7.2 Measuring mechanical waves

Worked example: Try yourself 7.2.1

DISPLACEMENT–DISTANCE GRAPH

The displacement–distance graph below shows a snapshot of a transverse wave as it travels along a spring towards the right.



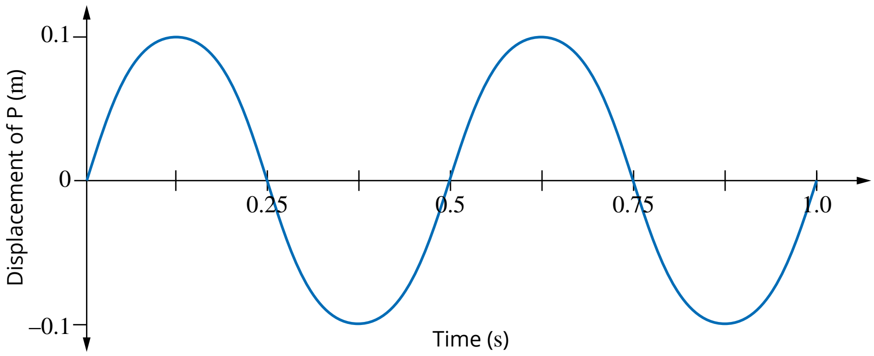
Use the graph to determine the amplitude, wavelength and wavenumber of this wave.

Thinking	Working
Amplitude on a displacement–distance graph is the distance from the average position to a crest or a trough. Read the displacement of a crest or a trough from the vertical axis. Convert to metres if necessary.	Amplitude = 2 cm = 0.02 m
Wavelength is the distance for one complete cycle. Any two consecutive points at the same position on the wave could be used.	The first cycle runs from the origin to intersect the horizontal axis at 0.4 m. This intersection is the wavelength. The wavelength is 0.4 m.
Wavenumber can be calculated using: $k = \frac{2\pi}{\lambda}$	$k = \frac{2\pi}{\lambda} = \frac{6.3}{0.4} = 15.7\ \text{m}^{-1}$

Worked example: Try yourself 7.2.2

DISPLACEMENT–TIME GRAPHS

The displacement–time graph below shows the motion of a single part of a rope as a wave passes travelling to the right.



Use the graph to find the amplitude, period, and frequency of the wave.

Thinking	Working
The amplitude on a displacement–time graph is the displacement from the average position to a crest or trough. Note the displacement of successive crests and/or troughs on the wave and carefully note units on the vertical axis.	Maximum displacement is 0.1 m. Therefore amplitude = 0.1 m.
Period on a displacement–time graph is the time it takes to complete one cycle and can be identified in the graph as the time between two successive points on the graph that are in phase. Identify two points on the graph at the same position in the wave cycle, e.g. the origin and $t = 0.5$ s. Confirm by checking two other points, e.g. two crests or two troughs.	Period $T = 0.5$ s.
Frequency can be calculated using $f = \frac{1}{T}$, measured in hertz (Hz).	$f = \frac{1}{T} = \frac{1}{0.5} = 2$ Hz

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Worked example: Try yourself 7.2.3

THE WAVE EQUATION

What is the frequency of a transverse wave that has a wavelength of 4.0×10^{-7} m and a speed of $3.0 \times 10^{8}\,\mathrm{m\,s^{-1}}$?

Thinking	Working
The wave equation states that $v = f\lambda$. Knowing both v and λ, the frequency f can be found. Rewrite the wave equation in terms of f.	$v = f\lambda$ $f = \frac{v}{\lambda}$
Substitute the known values and solve.	$f = \frac{v}{\lambda}$ $= \frac{3.0\times10^{8}}{4.0\times10^{-7}}$ $= 7.5 \times 10^{14}$ Hz

Worked example: Try yourself 7.2.4

THE WAVE EQUATION

Calculate the period of a transverse wave that has a wavelength of 4.0×10^{-7} m and a speed of $3.0 \times 10^{8}\,\mathrm{m\,s^{-1}}$.

Thinking	Working
Rewrite the wave equation in terms of T.	$v = f\lambda$ and $f = \frac{1}{T}$ $v = \frac{1}{T} \times \lambda$ $= \frac{\lambda}{T}$ $T = \frac{\lambda}{v}$
Substitute the known values and solve.	$T = \frac{\lambda}{v}$ $= \frac{4.0\times10^{-7}}{3.0\times10^{8}}$ $= 1.3 \times 10^{-15}$ s

7.2 Review

1 **a** C and F are in phase. A and C, and E and G, have the same displacements but are at different positions on the wave, so they are not in phase.
b Wavelength. Two points in phase are exactly one wavelength apart.
c B and D. Maximum displacement can be either positive or negative.
d Amplitude.

2 Wavelength is the length of one complete wave cycle. Any two points at the same position on the wave could be used. In this case $\lambda = 1.6$ m.
Wavenumber can be calculated using the formula: $k = \frac{2\pi}{\lambda} = \frac{6.3}{1.6} = 3.9\,\mathrm{m^{-1}}$
Amplitude is the displacement from the average position to a crest or trough.
In this case, amplitude = 20 cm = 0.2 m.

3 **a** Period = 0.4 s. This is the time for one cycle of the wave to occur.
b $f = \frac{1}{T} = \frac{1}{0.4} = 2.5$ Hz

4 $f = 5$ Hz, $A = 0.3$ m, $\lambda = 1.3$ m, $v = ?$
$v = f\lambda = 5 \times 1.3 = 6.5\,\mathrm{m\,s^{-1}}$

5 **a** True. This can be seen by rearranging $v = f\lambda$ to $\lambda = \frac{v}{f}$.
b False. The period of a wave is proportional to its wavelength.
c True. There is no equation that relates A to v.
d False. The wavelength and frequency of a wave determine its speed.

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6 a $\lambda = 4\,cm$; $A = 0.5\,cm$

b $T = 2\,s$, $\lambda = 4\,cm$, $v = ?$

$v = \frac{\lambda}{T} = \frac{4}{2} = 2\,cm\,s^{-1}$ or $0.02\,m\,s^{-1}$

c The red particle.

7 $T = \frac{1}{f} = \frac{1}{2\times10^5} = 5 \times 10^{-6}\,s$

CHAPTER 7 REVIEW

1 The particles on the surface of the water move up and down as the waves radiate outwards, carrying energy away from the point on the surface of the water where the stone entered the water.

2 Similarities: both are waves, both carry energy away from the source, both are caused by vibrations.

Differences: transverse waves involve particle displacement at right angles to the direction of energy travel of the wave; longitudinal waves involve particle displacement parallel to the direction of the wave energy.

3 Sound waves are longitudinal mechanical waves where the particles only move back and forth around an equilibrium position, parallel to the direction the wave energy travels. When these particles move in the direction of the wave, they collide with adjacent particles and transfer energy to the particles in front of them. This means that kinetic energy is transferred between particles in the same direction through collisions. Therefore, the particles cannot move along with the wave from the source as they lose their kinetic energy to the particles in front of them during the collisions.

4 In a transverse wave the motion of the particles is at right angles (perpendicular) to the direction of travel of the wave itself.

5 Longitudinal: a and d

Transverse: b and c

6 Mechanical waves move energy via the interaction of particles. The molecules in a solid are closer together than those in a gas. A smaller movement is needed to transfer energy, so the energy of the wave is usually transferred more quickly in a solid when compared with other states of matter.

7 The energy travels towards the left, that is, the energy is transferred away from the stone towards X.

8 U is moving down and V is momentarily stationary (and will then move downwards).

9 Period is the time it takes to complete one cycle. Period $T = 40\,ms = 40 \times 10^{-3}\,s = 4 \times 10^{-2}\,s$

Frequency can be calculated using $f = \frac{1}{T} = \frac{1}{4\times10^{-2}} = 25\,Hz$

10 Period can be calculated using $T = \frac{1}{f} = \frac{1}{200} = 0.005\,s = 5\,ms$

11 Five successive crests covers a distance of four wavelengths. The average wavelength will be $\frac{20}{4} = 5\,cm$.

12 The total distance from crest to trough is twice the amplitude. The amplitude of wave A is 2 units. The amplitude of wave B is 1 unit.

The ratio of the amplitudes = amplitude A ÷ amplitude B = 2.

13 The wavelength of wave A is 4 units. The wavelength of wave B is 2 units.

The ratio of the wavelengths = wavelength A ÷ wavelength B = 2.

14 Points are in phase if they have the same displacement from their rest position and they are moving in the same direction. Points B and D are in phase with each other. A and C are both moving in the same direction but are not at the same displacement, hence they are not in phase.

15 $f = 10.0\,Hz$, $\lambda = 30.0\,mm = 0.0300\,m$, $v = ?$

$v = f\lambda = 10 \times 0.03 = 0.300\,m\,s^{-1}$

16 $f = 22\,000\,Hz$, $v = 1200\,m\,s^{-1}$, $\lambda = ?$

$v = f\lambda$ rearranges to $\lambda = \frac{v}{f}$

$\lambda = 1200 \div 22\,000 = 0.055\,m$

17 $v = 1500\,m\,s^{-1}$, $f = 300\,Hz$, $\lambda = ?$

$v = f\lambda$ rearranges to $\lambda = \frac{v}{f}$

$\lambda = 1500 \div 300 = 5\,m$

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18 $v = 340\,\mathrm{m\,s^{-1}}$, $f = 300\,\mathrm{Hz}$, $\lambda = ?$

$v = f\lambda$ rearranges to $\lambda = \frac{v}{f}$

$\lambda = 340 \div 300 = 1.1\,\mathrm{m}$

19 Frequency is inversely proportional to wavelength.

The ratio of the frequency of A to the frequency of B is calculated as $\frac{250}{1000}$

Therefore, the ratio of wavelength A to the wavelength of B is $\frac{1000}{250} = 4$

20 From the wave equation $v = f\lambda$ since, if frequency decreases and the velocity must stay the same, the wavelength must increase. This ensures the product of the wavelength and frequency still equals the velocity, which has remained unchanged. (Note: Velocity is constant because it is a property of the medium.)

21 Responses will vary.

Waves, both mechanical and electromagnetic, can be quantified using the variables of: frequency, wavelength and speed, using the wave equation, $v = f\lambda$.

Mechanical waves require a medium to travel in while electromagnetic waves (such as visible light), do not need a medium.

The mechanical sound waves created by the tuning fork create the water waves which you have analysed. It was possible to find the inverse relationship between frequency and wavelength in this activity.

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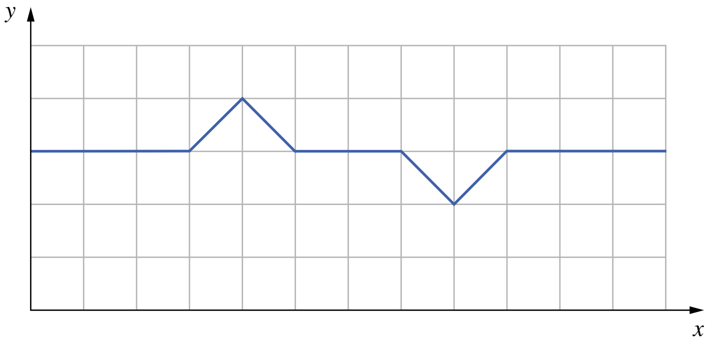
Chapter 8 Wave behaviour

8.1 Wave interactions

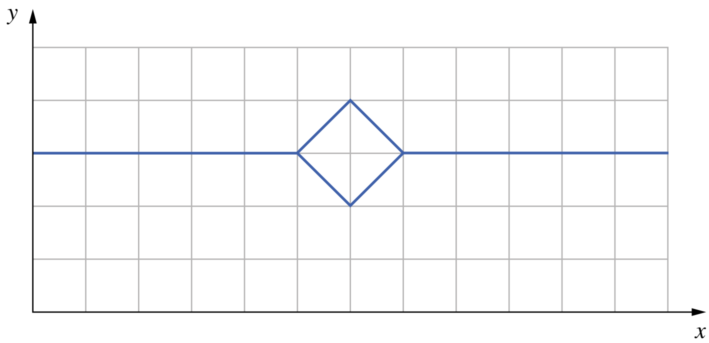
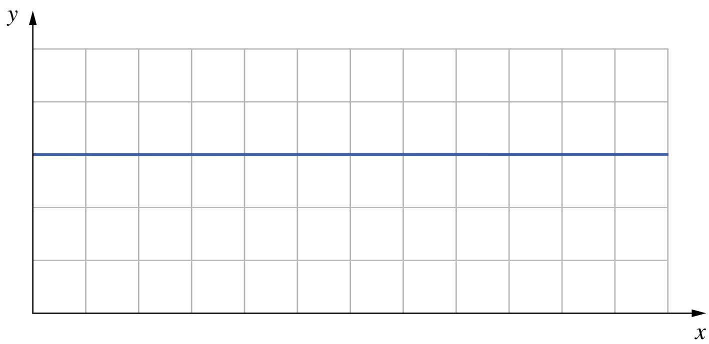
Worked example: Try yourself 8.1.1

WAVE SUPERPOSITION

Two wave pulses are travelling towards each other at $1\,m\,s^{-1}$, as shown in the figure below.



What is the amplitude of the combined pulse when they pass each other in 2 seconds?

Thinking	Working
Draw a diagram of the two pulses after 2 seconds.	
Draw a new diagram with the waves superimposed.	
Calculate the height of the resultant wave.	The amplitude of the resultant wave is 0.

8.1 KEY QUESTIONS

1 The wave is reflected and there is a 180° change in phase.

2 Since a wave is reflected back into the same medium, the only property that changes is amplitude. This is because some of the energy of the wave has been absorbed by the second medium that reflects the wave. (Note: Velocity changes in direction but speed does not change, because it is a scalar quantity.)

3 B. Each pulse travels 3 m in 3 s. Adding their amplitudes together means they will look like C, with the result that they cancel each other out, as in B.

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4 $i = 90° - 38° = 52°$

$r = i = 52°$

5 C. The object must have been convex, that is, curved outwards.

6 **a** Waves refract towards the normal when they slow down. The speed of the wave in medium A is faster than the speed of the wave in medium B.

b Waves cannot be gained or lost. There is the same number of waves. The frequency of the wave in medium A is the same as in B.

7 Increasing the frequency of the wave decreases its wavelength. Because significant diffraction occurs when $\frac{\lambda}{w} \geq 1$, as the wavelength decreases, the diffraction through the gap becomes less significant.

8.2 Resonance

8.2 KEY QUESTIONS

1 An object subjected to forces varying with its natural oscillating frequency will oscillate with increasing amplitude. This could continue until the structure can no longer withstand the internal forces and it fails.

2 B. If maximum energy is transferred, the amplitude increases. The frequency doesn't change.

3 Pendulum D would oscillate with the largest amplitude, as it has the same length as the pendulum on the left. This means that the natural frequency of pendulum D is equal to the driving frequency of the pendulum on the left, and the maximum energy is transferred to D.

4 The frequency of the idling motor of the truck (100 Hz), which is the forcing frequency in this situation, must be equal to (or close to) the natural frequency of vibration of the mirror, so that maximum energy is transferred to the mirror and it vibrates with large amplitude. When the truck is driving, the frequency of the motor is increased and it is no longer equal to the natural frequency of vibration of the mirror, so the amplitude of the vibrations is much smaller.

5 Normal walking results in a frequency of 1 Hz or 1 cycle (i.e. two steps) per second. This frequency may result in an increase in the amplitude of oscillation of the bridge over time, which could damage it.

CHAPTER 8 REVIEW

1 At the fixed end of a string or rope, a wave undergoes a phase change.

2 **a** transmission

b reflection

c absorption.

3 **a** True. When waves interact the individual displacements are added together to give a superposition.

b False. As the pulses pass through each other, they interact by either destructive or constructive superposition. Once the pulses have passed each other they return to their original shape.

c True.

4 A. The waves are bending slightly around a barrier.

5 **a** incident ray

b normal

c reflected ray

d boundary between media

e refracted ray

6 The bass sounds have larger wavelengths. Larger wavelength sounds are able to diffract more readily and can therefore spread out around obstacles and corners, making them easier to hear.

7 As light passes from water into air its speed increases, and it refracts away from the normal.

8 When viewed from directly above, the light ray strikes the interface between the air and water at an angle of incidence of 0°, and no refraction occurs. The fish will be observed at its actual position. That is, it will seem to be deeper in the water than in the previous question.

9 The wavelength of light is very small, in the order of 10^{-7} m. Significant diffraction only occurs when $\frac{\lambda}{w} \geq 1$. The wavelength of light is too small to diffract through and around everyday objects.

10 Reflection and refraction. At the boundary between the two mediums (hot and warm air) the sound is both reflected down and refracted through.

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11 The speed of sound in the hotter air is faster, as the sound wave has bent away from the normal.

12 C. Refraction or reflection of a wave does not change its frequency. Therefore the pitch of the sound is the same for the people on top of the building and those on the ground floor.

13 If there is no temperature difference between the two bodies of air, there will be no reflection and no refraction and therefore the people in the building may find that the sound from rock concert is quieter.

14 The green wave represents the superposition of the blue and the purple waves.

15 The two wave pulses must have the same frequency (or wavelength) and the same amplitude, and they are out of phase by 180°.

16 All objects/materials have a natural or resonant frequency. If the object is made to vibrate at this by applying an outside, driving frequency, the amplitude of the object's vibrations will increase with time. The amplitude of vibration will increase as the maximum possible energy is transferred to the resonating object. An example of resonance may be in musical instruments.

17 D. Significant diffraction occurs when $\frac{\lambda}{w}$ is approximately 1 or greater; $700\,\text{nm} \approx 1 \times 10^{-6}\,\text{m}$ and $0.001\,\text{mm} = 0.001 \times 10^{-3}$ or $10^{-6}\,\text{m}$.

18 It is a common misconception that standing waves somehow remain stationary. It is only the pattern made by the amplitude along the rope that stays still at certain points on the rope. The rope is still moving, with waves traveling up and down the rope, superimposing but giving the appearance of a wave 'standing still' at one place, with parts of the rope oscillating up and down.

19 The natural frequency of the human ear canal is approximately 2500 Hz. Since the driving frequency of the signal generator (2500 Hz) is the same as the natural frequency of the ear canal, there is maximum transfer of energy from the signal generator to the ear, which is perceived as a very loud sound.

20 All of the options are correct. The light rays striking all of these surfaces will obey the law of reflection, which always holds regardless of the shape of the reflector.

21 Responses will vary.

As waves interact with an object, they will cause it to vibrate at certain frequencies. Objects tend to vibrate at a certain frequency known as their resonant (or natural) frequency. If an outside source, such as the pull on the tin, causes the object to vibrate at its natural frequency, resonance will occur. This causes a maximum transfer of energy. This is shown in the activity as a maximum amplitude of displacement.

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Chapter 9 Sound waves

9.1 Sound as a wave

9.1 KEY QUESTIONS

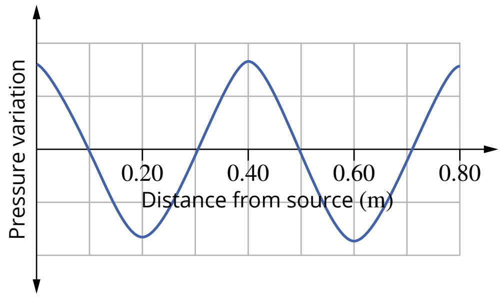
1 B. Sound is a mechanical wave form, energy is transferred by the molecules within a medium.

2 A and D are true. B is not true, as a wave transfers energy without permanently moving the particles in the medium. C is not true, as sound is a longitudinal wave (i.e. the motion is parallel to the direction of energy).

3 **a** E. The air molecules are vibrating left and right, as sound is a longitudinal wave.

b From the scale, the distance from compression at P to compression at R is 0.40 m.

c



4 **a** Compressions are regions where there is maximum pressure variation, with peaks as shown on the graph. Therefore, compressions pass through point X at $t = 0.5, 2.5, 4.5$ ms.

b $f = \frac{1}{T}$. Period is 2.0 ms or 2.0×10^{-3} s.

$\therefore f = \frac{1}{2.0\times10^{-3}} = 500\,\text{Hz}$

9.2 Sound behaviour

Worked example: Try yourself 9.2.1

THE INVERSE SQUARE LAW AND INTENSITY

The sound from a drill is 10 mW m^{-2} at a distance of 1.0 m. Assume that the sound spreads out equally in all directions. Calculate the intensity at a distance of 10 m.

Thinking	Working
Write out the inverse square law.	$I \propto \frac{1}{r^2}$
Create a relationship which compares the two intensities (I_1 and I_2) and the two distances (r_1 and r_2).	$I_1 \propto \frac{1}{r_1^2}$ $I_2 \propto \frac{1}{r_2^2}$ $\frac{I_1}{I_2} \propto \frac{r_2^2}{r_1^2}$
Substitute in the known values and solve for r_1.	$\frac{I_1}{I_2} \propto \frac{r_2^2}{r_1^2}$ $\frac{1.0\times10^{-2}}{I_2} = \frac{10^2}{1.0^2}$ $\therefore I_2 = \frac{1.0\times10^{-2}}{100}$ $= 1.0 \times 10^{-4}\,\text{W m}^{-2}$

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Worked example: Try yourself 9.2.2

THE DOPPLER EFFECT

An aeroplane is flying at 150 m s^{-1} overhead and is emitting a constant sound of frequency 1000 Hz. What frequency sound would a stationary observer hear once the aeroplane has passed? Take the speed of sound in air to be 340 m s^{-1}.

Thinking	Working
Determine the variables required to use the Doppler effect formula.	f = original frequency of the sound = 1000 Hz v_{wave} = speed of the waves in the medium = 340 m s^{-1} $v_{observer}$ = speed of the observer relative to the medium = 0 v_{source} = the speed of the source relative to the medium = −150 m s^{-1} (i.e. moving away from the observer)
Calculate the frequency as heard by the observer.	$f' = 1000\left(\frac{340+0}{340+150}\right)$ $= 1000 \times (0.6939)$ $= 693.9$ $= 694$ Hz The observer hears a lower-pitch sound of 694 Hz.

Worked example: Try yourself 9.2.3

BEAT FREQUENCY

In the physics laboratory, a student is testing two tuning forks. One is labelled at 349 Hz, and the other is unlabelled. When struck together, the two tuning forks produce a beat frequency of 4 Hz. What are the possible frequencies of the second tuning fork?

Thinking	Working
Write out the formula for the beat frequency.	$f_{beat} = \lvert f_1 - f_2 \rvert$
Substitute the values for the different frequencies.	$f_{beat} = \lvert f_1 - f_2 \rvert$ $4 = \lvert 349 - f_2 \rvert$ $4 = \lvert 349 - 345 \rvert$ or $4 = \lvert 349 - 353 \rvert$ $f_2 = 345$ Hz or 353 Hz

9.2 KEY QUESTIONS

1 Turning up the volume increases energy and therefore the amplitude.

2 $\frac{I_1}{I_2} = \frac{r_2^2}{r_1^2}$, $\frac{0.01}{I_2} = \frac{1.0}{10.0^2}$, $I_2 = \frac{0.01 \times 100}{1} = 1.0\,\text{W m}^{-2}$. This is equal to the threshold of pain for people.

3 As the speed of each vehicle is the same and there is no relative motion of the medium, the frequency observed would be the same as that at the source.

4 $f' = f\left(\frac{v_{wave} + v_{observer}}{v_{wave} - v_{source}}\right) = 100\left(\frac{340+0}{340-8}\right)$, $f' = 102.4 = 102$ Hz

5 Three possible beat frequencies can be produced:

$f_{beat} = \lvert f_1 - f_2 \rvert$, $f_{beat} = \lvert 440 - 438 \rvert = 2$ Hz, or $f_{beat} = \lvert 440 - 441 \rvert = 1$ Hz,

or $f_{beat} = \lvert 441 - 438 \rvert = 3$ Hz

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9.3 Standing waves

Worked example: Try yourself 9.3.1

FUNDAMENTAL FREQUENCY IN STRINGS

A 25 cm length of string is fixed at both ends. Assume that the tension of the string is not changed.

a What is the wavelength of the fundamental frequency?	
Thinking	**Working**
Identify the length of the string (l) in metres and the harmonic number (n).	$l = 25\text{ cm} = 0.25\text{ m}$ $n = 1$
Recall that for any frequency, $\lambda = \frac{2l}{n}$. Substitute the values from the question and solve for λ.	$\lambda = \frac{2l}{n}$ $= \frac{2 \times 0.25}{1} = 0.50\text{ m}$

b What is the wavelength of the third harmonic?	
Thinking	**Working**
Identify the length of the string (l) in metres and the harmonic number (n).	$l = 0.25\text{ m}$ $n = 3$
Recall that for any frequency, $\lambda = \frac{2l}{n}$. Substitute the values from the question and solve for λ.	$\lambda = \frac{2l}{n}$ $= \frac{2 \times 0.25}{3} = 0.17\text{ m}$

Worked example: Try yourself 9.3.2

HARMONIC WAVELENGTHS IN OPEN-ENDED AIR COLUMNS

A small flute (a piccolo) has a length of 32 cm and the sound has a speed of 320 m s^{-1}. Consider the piccolo to be an 'open-ended' pipe.

a What is the wavelength of the fourth harmonic?	
Thinking	**Working**
Identify the length of the air column (l) in metres and the harmonic number (n).	$l = 32\text{ cm} = 0.32\text{ m}$ $n = 4$
Recall that for any wavelength, $\lambda = \frac{2l}{n}$. Substitute the values from the question and solve for λ.	$\lambda_n = \frac{2l}{n}$ $= \frac{2 \times 0.32}{4}$ $= 0.16\text{ m}$

b What is the frequency of the fourth harmonic?	
Thinking	**Working**
Identify the wavelength (λ) of the note in metres and the speed of sound.	$\lambda = 0.16\text{ m}$ $v = 320\text{ m s}^{-1}$
Use the values in the wave equation, $v = f\lambda$	$f = \frac{v}{\lambda}$ $= \frac{320}{0.16}$ $= 2000\text{ Hz, or } 2.0\text{ kHz}$

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Worked example: Try yourself 9.3.3

FUNDAMENTAL FREQUENCY IN CLOSED AIR COLUMNS

An organ pipe can produce a fundamental frequency of 126 Hz. The speed of sound in the pipe is 340 m s^{-1}. Organ pipes are closed at one end.

a What is the length of the pipe? Give your answer in centimetres.	
Thinking	**Working**
Identify the frequency (f) in Hertz, the speed (v) in metres and the harmonic number (n).	$f = 126$ Hz $n = 1$
Recall the formula for frequency. Substitute the values from the question and solve for l.	$f = \frac{nv}{4l}$ $126 = \frac{1 \times 340}{4l}$ $l = \frac{340}{4 \times 126}$ $= 0.675$ m $= 67.5$ cm

b What is the wavelength of the next possible harmonic that can be produced in this pipe?	
Thinking	**Working**
Identify the length of the pipe (l) in metres and the harmonic number (n).	$l = 0.675$ m $n = 3$, because only odd harmonics are possible for a closed-end pipe.
Recall the formula for any wavelength. Substitute the values from the question and solve for λ.	$\lambda = \frac{4l}{n}$ $= \frac{4 \times 0.675}{3}$ $= 0.900$ m

9.3 KEY QUESTIONS

1 Wavelength of fundamental:
$\lambda = \frac{2l}{n} = \frac{2 \times 0.4}{1} = 0.8$ m

2 Rearrange $\lambda = \frac{2l}{n}$ to give the length of the string:
$l = \frac{n\lambda}{2} = \frac{4 \times 0.75}{2} = 1.5$ m

3 This wave has a frequency four times that of the fundamental frequency.Therefore it has a wavelength $\frac{1}{4}$ of the fundamental wavelength, due to the inverse relationship between frequency and wavelength.

4 The length of the violin string:
$l = \frac{nv}{2f} = \frac{1 \times 387}{2 \times 350} = 0.55$ m
New length $= \frac{2}{3} \times 0.55 = 0.37$ m.

Wavelength of the new fundamental:

$\lambda = 2 \times$ new length $= 2 \times 0.37 = 0.74$ m

5 **a** Fundamental frequency of string: $f = \frac{nv}{2l} = \frac{1 \times 300}{2 \times 0.5} = 300$ Hz

b Frequency of second harmonic: $f = \frac{nv}{2l} = \frac{2 \times 300}{2 \times 0.5} = 600$ Hz

c Frequency of third harmonic: $f = \frac{nv}{2l} = \frac{3 \times 300}{2 \times 0.5} = 900$ Hz

6 **a** Wavelength of second harmonic: $\lambda = \frac{2l}{2} = 50$ cm or 0.50 m

b Wavelength of third harmonic: $\lambda = \frac{2l}{3} = 33.3$ cm or 0.33 m

c Frequency of third harmonic: $f = \frac{v}{\lambda} = \frac{320}{0.33} = 960$ Hz

7 **a** Fundamental frequency of pipe: $f_1 = \frac{v}{\lambda} = \frac{315}{4l} = \frac{315}{2.8} = 112.5$ or 113 Hz

b Frequency of fifth harmonic: $f_5 = 5 \times f_1 = 562.5$ or 563 Hz

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CHAPTER 9 REVIEW

1 B. The amplitude of A is 2.5 and the amplitude of B is 4.

2 C. From horizontal scale of graphs supplied. The period of A is 4 units long while the period of B is 2 units long. Period is inversely proportional to frequency.

3 Graph B depicts the higher pitched sound because it has a higher frequency.

4 Graph B depicts a louder sound as it has a greater amplitude.

5 B and C. Pressure variation is a maximum at a compression and minimum at a rarefaction.

6 A. The amplitude is equal to the maximum displacement from the mean (i.e. at a peak or a trough).

7 C. One complete wave along the horizontal scale.

8 D. The sound is a longitudinal wave moving to the right. At point Y, the particle is at maximum negative displacement, which means it is currently as far left as it can go. Therefore, it is about to be moving right.

9 A, B and D. Speed is determined by the medium.

10 C and D. The pitch changed because of the Doppler effect. Because the question does not say whether the pitch became higher or lower, the motorbike could have been travelling away from you and then travelled towards you, or travelling towards you and then travelled away from you.

11 **a** Convert $180\,\text{km}\,\text{h}^{-1}$ in to $\text{m}\,\text{s}^{-1}$: $180\,\text{km}\,\text{h}^{-1} = 50\,\text{m}\,\text{s}^{-1}$:

$$f' = f\left(\frac{v_{\text{wave}} + v_{\text{observer}}}{v_{\text{wave}} - v_{\text{source}}}\right) = 2000\left(\frac{340+0}{340-50}\right) = 2000\left(\frac{340}{290}\right) = 2.34\,\text{kHz}$$

b Convert $180\,\text{km}\,\text{h}^{-1}$ in to $\text{m}\,\text{s}^{-1}$: $180\,\text{km}\,\text{h}^{-1} = 50\,\text{m}\,\text{s}^{-1}$:

$$f' = f\left(\frac{v_{\text{wave}} + v_{\text{observer}}}{v_{\text{wave}} + v_{\text{source}}}\right) = 2000\left(\frac{340+0}{340+50}\right) = 2000\left(\frac{340}{390}\right) = 1.74\,\text{kHz}$$

12 For a wave that is propagated in a medium, relative motion between the source, observer and medium can all cause the Doppler effect.

13 $\frac{I_1}{I_2} = \frac{r_1^2}{r_2^2}$

$$\therefore I_2 = \frac{2.2\times10^{-5}\times3^2}{8^2} = 0.31\times10^{-5} = 3.1\ \mu\text{W}\,\text{m}^{-2}$$

14 **a** $f = \frac{nv}{4l} = \frac{1\times340}{4\times0.85} = 100\,\text{Hz}$

b $f = \frac{nv}{4l} = \frac{3\times340}{4\times0.85} = 300\,\text{Hz}$

15 The fundamental frequency is given by $f_1 = \frac{1}{t} = \frac{1}{4} = 0.25\,\text{Hz}$.

The frequency of the second harmonic is $f_2 = 2 \times f_1 = 0.50\,\text{Hz}$.

16 $\lambda = \frac{v}{f} = \frac{78}{428} = 0.182\,\text{m}$.

The separation, d, of antinodes and of nodes in a standing wave in a string with fixed ends is half the wavelength:

$\therefore d = \frac{\lambda}{2} = \frac{0.182}{2} = 0.091\,\text{m}$ or $9.1\,\text{cm}$

17 **a** $f = \frac{nv}{4l} = \frac{1\times320}{4\times0.64} = 125\,\text{Hz}$

b $\lambda = \frac{4l}{n} = \frac{4\times0.64}{1} = 2.56\,\text{m}$

c $f = \frac{nv}{4l} = \frac{3\times320}{4\times0.64} = 375\,\text{Hz}$

(An alternative method is: $f_3 = 3 \times f_1 = 3 \times 125 = 375\,\text{Hz}$)

18 The frequency of sound in strings is dependent on the mass of the string. Guitar strings change mass as they become thicker, so each string can produce a different frequency.

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19 $f = \frac{v}{2l}$

$200 = \frac{8}{2l}$

$l = \frac{8}{400}$

$= 0.02\,\text{m}$

$= 20\,\text{mm}$

20 Responses will vary. The behaviour of sound waves can be predicted using a wave model. The wave behaviours that you see in this inquiry activity are diffraction and superposition. As the waves leave the speaker it diffracts, forming a circle of sound waves around each speaker. In areas of loud noise, the waves from each speaker arrive in phase and constructively interfere. In areas of quiet noise the waves from each speaker arrive out of phase – a compression from one wave arrives with a rarefaction from the other, and vice versa. The sound does not completely cancel due to reflections and other small variations. This interference pattern is caused by the wave nature of sound.

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Chapter 10 Ray model of light

10.1 Light as a ray

Worked example: Try yourself 10.1.1

CALCULATING INTENSITY

The intensity of the light 50 cm away from an LED (i.e. light-emitting diode) is 0.016 cd. Calculate the intensity of the light at a distance of 10 cm from the LED.

Thinking	Working
Recall the formula for intensity.	$I_1 r_1^2 = I_2 r_2^2$
Rearrange the formula to make I_2 the subject.	$I_2 = \frac{I_1 r_1^2}{r_2^2}$
Substitute the appropriate values into the formula and solve.	$I_2 = \frac{0.016 \times 0.5^2}{0.1^2} = 0.4\ \text{cd}$

Worked example: Try yourself 10.1.2

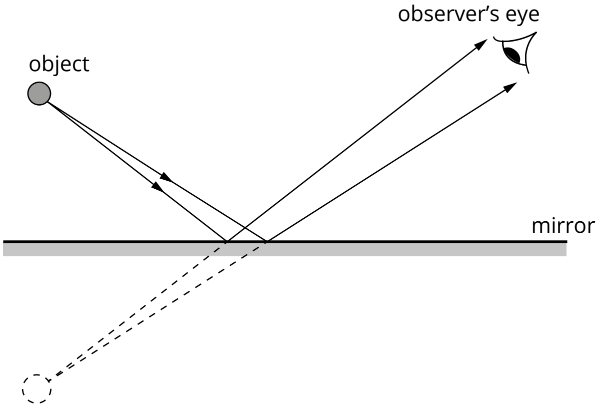
CALCULATING INTENSITY CHANGES USING PROPORTIONAL REASONING

Calculate the change in the intensity of the light from an LED (light-emitting diode) if it is brought 5 times closer.

Thinking	Working
Recall the formula for intensity.	$I_1 r_1^2 = I_2 r_2^2$
Describe the change to radius as an equation.	$r_2 = \frac{1}{5} r_1$
Substitute this relationship into the formula for intensity.	$I_1 r_1^2 = I_2 \left(\frac{1}{5} r_1\right)^2$
Simplify this equation to describe the change in intensity.	$I_1 r_1^2 = I_2 \times \frac{1}{25} r_1^2$ $\therefore I_1 = I_2 \times \frac{1}{25}$ $\therefore I_2 = 25 I_1$
Interpret the answer.	The light will be 25 times brighter in the new position.

10.1 KEY QUESTIONS

1



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2 **a** upright
b the same size as the object
c virtual

3 **a** She must be 1.5 m away from the mirror to appear to be 3 m from her image. She must walk 7 – 1.5 = 5.5 m.
b Since she and her image both move at 1.5 m s^{-1} in opposite directions, their combined relative speed is 3.0 m s^{-1}.

4 When you read a book, light from your face is reflected off the page in front of you. However, since the surface of the page is rough, the reflection is diffuse—the light rays travel in random directions and do not form an image.

5 D.

$$r_2 = 2.5 \times r_1$$
$$I_1 {r_1}^2 = I_2 (2.5 r_1)^2$$
$$I_1 {r_1}^2 = I_2 \times 6.25 {r_1}^2$$
$$I_2 = \frac{1}{6.25} I_1$$

6 $I_2 = \frac{I_1 r_1^2}{r_2^2}$

$$= \frac{6.0 \times 0.75^2}{0.5^2} = 13.5 \text{ cd}$$

10.2 Refraction

Worked example: Try yourself 10.2.1

CALCULATING REFRACTIVE INDEX

The speed of light in crown glass (a type of glass used in optics) is 1.97×10^8 m s^{-1}.
Given that the speed of light in a vacuum is 3.00×10^8 m s^{-1}, calculate the refractive index of crown glass.

Thinking	Working
Recall the definition of refractive index.	$n_{\text{crown glass}} = \frac{c}{v_{\text{crown glass}}}$
Substitute the appropriate values into the formula and solve.	$n_{\text{crown glass}} = \frac{3.00 \times 10^8}{1.97 \times 10^8} = \frac{3.00}{1.97}$ $= 1.52$

Worked example: Try yourself 10.2.2

CHANGES TO THE SPEED OF LIGHT

A ray of light travels from water (n = 1.33), where it has a speed of 2.25×10^8 m s^{-1}, into flint glass (n = 1.85).
Calculate the speed of light in flint glass.

Thinking	Working
Recall the formula.	$n_1 v_1 = n_2 v_2$
Substitute the appropriate values into the formula and solve.	$1.33 \times 2.25 \times 10^8 = 1.85 \times v_2$ $v_2 = \frac{1.33 \times 2.25 \times 10^8}{1.85}$ $= 1.62 \times 10^8$ m s^{-1}

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Worked example: Try yourself 10.2.3

USING SNELL'S LAW

A ray of light in air strikes a piece of flint glass (n = 1.62) at 50° to the normal. Calculate the angle of refraction of the light in the glass.

Thinking	Working
Recall Snell's law.	$n_1 \sin i = n_2 \sin r$
Recall the refractive index of air.	$n_1 = 1.00$
Substitute the appropriate values into the formula to find a value for sin *r*.	$1.00 \times \sin 50° = 1.62 \times \sin r$ $\sin r = 0.4729$
Calculate the angle of refraction.	$r = \sin^{-1} 0.4729$ $= 28.2°$

Worked example: Try yourself 10.2.4

CALCULATING CRITICAL ANGLE

Calculate the critical angle for light passing from diamond (n = 2.42) into air.

Thinking	Working
Recall the equation for the critical angle.	$\sin i_c = \frac{1}{n_x}$
Substitute the refractive indexes of diamond and air into the formula.	$\sin i_c = \frac{1.00}{2.42} = 0.4132$
Solve for i_c.	$i_c = \sin^{-1} 0.4132$ $= 24.4°$

10.2 KEY QUESTIONS

1 Therefore, the speed of light in seawater is *slower than* in pure water.

2 Recall the definition of refractive index: $n_{\text{seawater}} = \frac{c}{v_{\text{seawater}}}$
Rearrange to get

$$\begin{aligned} v_{\text{seawater}} &= \frac{c}{n_{\text{seawater}}} \\ &= \frac{3.00 \times 10^8}{1.38} \\ &= 2.17 \times 10^8 \text{ m s}^{-1} \end{aligned}$$

3 $n_1 v_1 = n_2 v_2$

$$\begin{aligned} 1.33 \times 2.25 \times 10^8 &= n_2 \times 2.29 \times 10^8 \\ \therefore n_2 &= 1.31 \end{aligned}$$

4 Recall Snell's law: $n_1 \sin i = n_2 \sin r$

$$1.33 \times \sin 44° = 1.60 \times \sin r$$

$$\begin{aligned} \therefore \sin r &= \frac{1.33 \times \sin 44°}{1.60} \\ &= 0.5774 \\ \therefore r &= \sin^{-1} 0.5774 \\ &= 35.3° \end{aligned}$$

5 Total internal reflection occurs when light passes from a medium with high refractive index into a medium with lower refractive index and refracts away from the normal.

a no
b yes
c yes
d no

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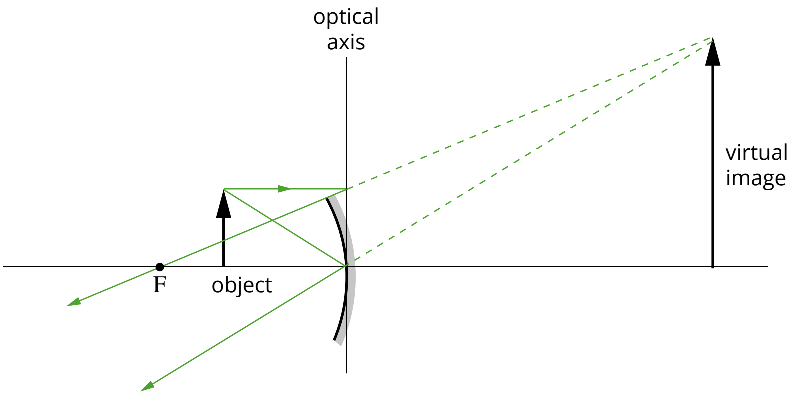
10.3 Curved mirrors and lenses

Worked example: Try yourself 10.3.1

LOCATING THE IMAGE FORMED BY A CONCAVE MIRROR

A dentist wishing to view a cavity in a tooth holds a concave mirror of focal length 12 mm at a distance of 8 mm from the tooth.

a Construct a ray diagram of this situation to describe the type of image formed.	
Thinking	**Working**
Construct the ray diagram. (Only two rays are needed.)	
Interpret the ray diagram.	The image will be virtual, upright and magnified.

b Use the mirror formula to locate the image.	
Thinking	**Working**
Recall the mirror formula.	$\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$
Substitute the information from the question into the formula.	$\frac{1}{12} = \frac{1}{8} + \frac{1}{v}$
Solve the equation.	$\frac{1}{v} = \frac{1}{12} - \frac{1}{8} = \frac{2}{24} - \frac{3}{24} = \frac{-1}{24}$ $\therefore\ v = -24$ mm
Interpret the answer. A negative answer for the image distance means that the image is virtual. A positive answer implies a real image.	Since the answer is negative, this confirms that the image will be a virtual image. It will appear to be located 24 mm behind the mirror.

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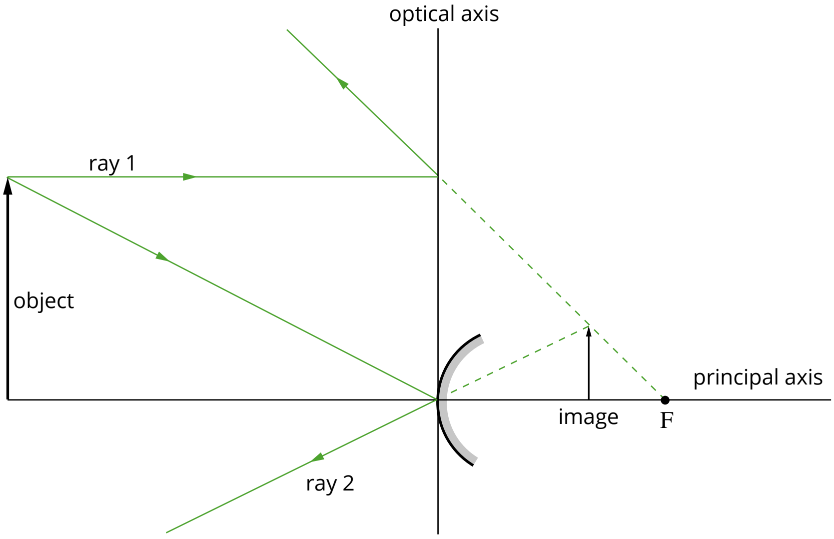
Worked example: Try yourself 10.3.2

LOCATING THE IMAGE FORMED BY A CONVEX MIRROR

An object is placed 30 cm in front of a convex mirror of focal length 15 cm. Determine the position of the image.

a Construct a ray diagram of this situation to describe the type of image formed.	
Thinking	**Working**
Construct the ray diagram. (Only two rays are needed.)	
Interpret the ray diagram.	The image is virtual, upright and diminished.

b Use the mirror formula to locate the image.	
Thinking	**Working**
Recall the mirror formula.	$\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$
Substitute the information from the question into the formula.	$\frac{1}{-15} = \frac{1}{30} + \frac{1}{v}$
Solve the equation.	$\frac{1}{v} = \frac{-1}{15} - \frac{1}{30} = \frac{-2}{30} - \frac{1}{30} = \frac{-3}{30} = \frac{-1}{10}$ $\therefore v = -10\text{ cm}$
Interpret the answer.	Since the answer is negative, this means that the image is a virtual image that appears to be located 10 cm behind the mirror.

Worked example: Try yourself 10.3.3

USING THE MAGNIFICATION FORMULA

An object 12 cm high is placed 30 cm in front of a convex mirror of focal length 15 cm. The mirror produces a virtual image 10 cm behind the mirror.

Calculate the magnification and height of the image.

Thinking	**Working**
Recall the magnification formula.	$M = \frac{h_i}{h_o} = \frac{v}{u}$
Substitute the information from the question into the formula.	$M = \frac{h_i}{12} = \frac{-10}{30}$ Note that because the image is virtual, its distance is given as a negative value.

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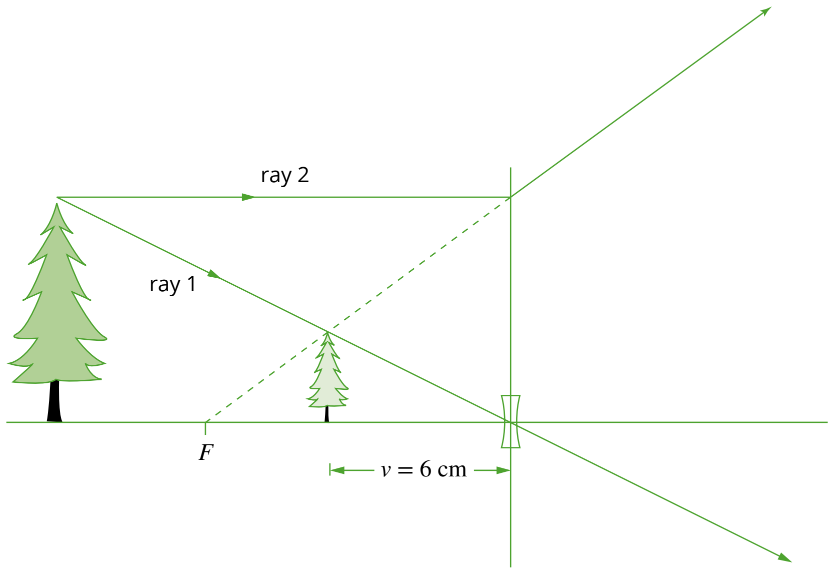
Solve the equation.	$M = -\frac{1}{3}$ and $h_i = -4$ cm
Interpret the answer.	The image is 3 times smaller than the object and so appears 4 cm high. (The negative values remind us that that the image is virtual.)

Worked example: Try yourself 10.3.4

USING THE LENS AND MAGNIFICATION FORMULAS

An artist uses a concave lens of focal length 10 cm to reduce a sketch of a tree to see what it would look like in miniature. The tree in the sketch 20 cm tall. The lens is held 15 cm above the page.

a Construct a ray diagram to describe the image.	
Thinking	**Working**
Construct the ray diagram. (Only two rays are needed.)	
Interpret the ray diagram.	The image is virtual, upright and diminished.

b Use the lens formula to determine the image distance and whether the image formed is real or virtual.	
Thinking	**Working**
Recall the lens formula.	$\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$
Substitute the information from the question into the formula.	Since the lens is a concave (i.e. diverging) lens, its focal length is given as a negative value. $\frac{1}{-10} = \frac{1}{15} + \frac{1}{v}$
Solve the equation.	$\frac{1}{v} = \frac{1}{-10} - \frac{1}{15} = -\frac{3}{30} - \frac{2}{30} = \frac{5}{30}$ $\therefore v = -\frac{30}{5} = -6\,\text{cm}$
Interpret the answer.	Since the answer is negative, the image is a virtual image. It appears 6 cm behind the lens.

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c Calculate the magnification of the image.	
Thinking	**Working**
Recall the magnification formula.	$M = \frac{h_i}{h_o} = \frac{v}{u}$
Substitute the information from the question into the formula.	$M = \frac{h_i}{20} = \frac{-6}{15}$
Solve the equation.	$M = -0.4$ and $h_i = 8\,\text{cm}$
Interpret the answer.	The image is reduced to two-fifths of its original size and so appears 8 cm high.

10.3 KEY QUESTIONS

1 **a** The radius of curvature describes the distance between the centre of a spherical mirror and the surface of the mirror.

b $f = \frac{1}{2}r = \frac{50}{2} = 25\,\text{cm}$

2 A concave mirror converges light rays. A convex mirror diverges light rays.

3 **a** virtual

b upright

c diminished

4 40 cm. An object that is twice the focal length away from a concave mirror forms an inverted image the same size as the object.

5 **a** $\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$

$\frac{1}{15} = \frac{1}{8} + \frac{1}{v}$

$\frac{1}{v} = \frac{1}{15} - \frac{1}{8} = \frac{8}{120} - \frac{15}{120} = -\frac{7}{120}$

$v = -\frac{120}{7}$

$= -17.1\text{ mm}$

Since the answer is negative, the image is virtual and upright.

b $M = \frac{h_i}{h_o} = \frac{v}{u}$

$\therefore h_i = \frac{v}{u} \times h_o = \frac{-17.1}{8} \times 2$

$= -4.3\text{ mm}$

The cavity appears as a virtual image 4.3 mm high.

6 20 cm. The image of a distant object is formed at the focal point of the lens.

7 D. Since the object is inside the focal length of the lens, the image will be virtual, upright and enlarged.

CHAPTER 10 REVIEW

1 The angle of incidence is equal to the angle of reflection, i.e. $i = r$.

2 A real image occurs where light rays actually cross and an image can be formed on a screen. A virtual image occurs where light rays appear to cross; it can be seen by looking into a mirror or lens.

3 **a** The image will be virtual, upright and the same size as the person.

b $2 \times 0.75\,\text{m} = 1.5\,\text{m}$

4 **a** 0.75 m. Using the law of reflection, a ray striking the mirror at this height will reflect back into the person’s eyes at 1.5 m above the ground.

b 1.55 m. The top of the person's head is 0.1 m above their eyes. Using the law of reflection, a ray from the top of the person's head into their eyes will hit the mirror 0.05 m above their eyes.

c 1.55 m – 0.75 m = 0.80 m. The mirror must be at least 80 cm long so that the rays from the person’s feet and the top of their head will both reach their eyes.

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5 Because the luminous intensity is proportional to $\frac{1}{r^2}$, decreasing the distance by half would mean the intensity would increase by a factor of four.

6 $I_2 = \frac{I_1 r_1^2}{r_2^2}$

$\therefore I_2 = \frac{5.0 \times 3.0^2}{4.5^2} = 2.2$ cd

7 As light travels from quartz ($n = 1.46$) to water ($n = 1.33$), its speed *increases* which causes it to refract *away from* the normal.

8 **a** incident ray

b normal

c reflected ray

d boundary between media

e refracted ray

9 $n_1 \sin i = n_2 \sin r$

$\sin(r) = \frac{n_{\text{glycerine}} \times \sin i}{n_{\text{water}}} = \frac{1.47 \times \sin 23.0}{1.33}$

$= 0.432$

$\therefore r = \sin^{-1} 0.432 = 25.6°$

10 $n_1 \sin i = n_2 \sin r$

$n_2 = \frac{n_1 \sin i}{\sin r} = \frac{1.00 \times \sin 43}{\sin 28.5} = 1.43$

$n_{\text{perspex}} = \frac{c}{v_{\text{perspex}}}$

$\therefore v_{\text{perspex}} = \frac{c}{n_{\text{perspex}}} = \frac{3.00 \times 10^8}{1.43} = 2.10 \times 10^8 \text{ m s}^{-1}$

11 Use Snell's law: $n_1 \sin i = n_2 \sin r$

For angle *a*:

$1.00 \times \sin 40 = 1.50 \times \sin a$

$\therefore \sin a = \frac{1.00 \times \sin 40}{1.5}$

$= 0.4285$

$a = \sin^{-1} 0.4285 = 25.4°$

For angle *b*:

Since *a* and *b* are corresponding angles, $a = b = 25.4°$

For angle *c*:

$1.50 \times \sin 25.4 = 1.33 \times \sin c$

$\therefore \sin c = \frac{1.50 \times \sin 25.4}{1.33}$

$= 0.4833$

$c = \sin^{-1} 0.4833 = 28.9°$

12 **a** The angle of incidence is measured with respect to the normal which is drawn at a right angle to the glass–air boundary.

$i = 90 - 58.0 = 32.0°$.

b $n_1 \sin i = n_2 \sin r$

$1.52 \times \sin 32 = 1.00 \times \sin r$

$\therefore \sin r = \frac{n_1 \sin i}{n_2} = \frac{1.52 \times \sin 32.0}{1.00}$

$= 0.8055$

$r = \sin^{-1} 0.8055 = 53.7°$

c $r - i = 53.7° - 32°$

$= 21.7°$

d $v_{\text{glass}} = \frac{c}{n_{\text{glass}}} = \frac{3 \times 10^8}{1.52}$

$= 1.97 \times 10^8 \text{ m s}^{-1}$

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13 a red light:

$n_1 \sin i = n_2 \sin r$

$1.00 \times \sin 30 = 1.50 \times \sin r_{red}$

$\therefore \sin r_{red} = \frac{1.00 \times \sin 30}{1.50}$

$= 0.3333$

$r_{red} = \sin^{-1} 0.3333 = 19.5°$

b violet light:

$n_1 \sin i = n_2 \sin r$

$1.00 \times \sin 30 = 1.53 \times \sin r_{violet}$

$\therefore \sin r_{violet} = \frac{1.00 \times \sin 30}{1.53}$

$= 0.3268$

$r_{violet} = \sin^{-1} 0.3268 = 19.1°$.

c $\Delta r = r_2 - r_1 = 19.5 - 19.1 = 0.4°$

d $v_{crown\ glass} = \frac{c}{n_{crown\ glass}} = \frac{3 \times 10^8}{1.53}$

$= 1.96 \times 10^8\,\mathrm{m\,s^{-1}}$

14 $\sin i_c = \frac{1}{n_x}$

$\therefore\ i_c = \sin^{-1}\left(\frac{1}{n_x}\right)$.

a $i_c = \sin^{-1}\left(\frac{1.00}{1.31}\right) = 49.8°$

b $i_c = \sin^{-1}\left(\frac{1.00}{1.54}\right) = 40.5°$

c $i_c = \sin^{-1}\left(\frac{1.00}{2.16}\right) = 27.6°$

15 B, D, A, C. The bigger the difference in refractive indexes, the bigger the angle of deviation. The air–water boundary has the smallest difference in refractive indices so it produces the smallest angle of deviation. The air–diamond boundary has the biggest difference in refractive indices so it produces the biggest angle of deviation.

16 Refractive index is defined as $n_x \geq \frac{c}{v_x}$.

Einstein's proposition means that $v_x \leq c$. Therefore $n_x \geq \frac{c}{c} = 1$.

No medium can have a refractive index below 1.

17 a 80 cm. An object that is twice the focal length away from a concave mirror forms an inverted image the same size as the object.

b Inside the focal length of the mirror, i.e. closer than 40 cm to the mirror.

18 a $\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$

$\frac{1}{-3} = \frac{1}{30} + \frac{1}{v}$

$\frac{1}{v} = \frac{1}{-3} - \frac{1}{30} = \frac{-10}{30} - \frac{1}{30} = \frac{-11}{30}$

$\therefore v = -\frac{30}{11} = -2.7$ m

Since the answer is negative, the image is virtual and upright and appears to be 2.7 m behind the mirror.

b $M = \frac{v}{u} = \frac{-2.7}{30} = 0.091$. The image is diminished.

19 a $\frac{1}{5} = \frac{1}{10} + \frac{1}{v}$

$\frac{1}{v} = \frac{1}{5} - \frac{1}{10} = \frac{2}{10} - \frac{1}{10} = \frac{1}{10}$

$\therefore v = 10$ cm

A real image is formed 10 cm in front of the mirror.

b $\frac{1}{5} = \frac{1}{5} + \frac{1}{v}$

$\frac{1}{v} = \frac{1}{5} - \frac{1}{5} = \frac{0}{1}$

No image would be formed (or the image would be formed at infinity) because the object is at the focal point.

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c $\frac{1}{5}=\frac{1}{2}+\frac{1}{v}$

$\frac{1}{v}=\frac{1}{5}-\frac{1}{2}=\frac{2}{10}-\frac{5}{10}=\frac{-3}{10}$

$\therefore v=-\frac{10}{3}=-3.3$ cm

A virtual image is formed 3.3 cm behind the mirror.

20 A concave lens always forms a virtual image because it causes rays to diverge.

21 a $\frac{1}{f}=\frac{1}{u}+\frac{1}{v}$

$\frac{1}{40}=\frac{1}{u}+\frac{1}{120}$

$\frac{1}{u}=\frac{1}{40}-\frac{1}{120}=\frac{3}{120}-\frac{1}{120}=\frac{2}{120}$

$\therefore u=\frac{120}{2}=60$ cm

The object was 60 cm from the lens.

b $h_i=\frac{v}{u}\times h_o=\frac{120}{60}\times 5=10\,\text{cm}$

The object is 10 cm high.

22 a $\frac{1}{f}=\frac{1}{u}+\frac{1}{v}$

$\frac{1}{15}=\frac{1}{10}+\frac{1}{v}$

$\frac{1}{v}=\frac{1}{15}-\frac{1}{10}=\frac{2}{30}-\frac{3}{30}=\frac{-1}{30}$

$\therefore v=-30\,\text{cm}$

The virtual image is formed 30 cm behind the lens.

b $M=\frac{v}{u}=\frac{-30}{10}=-3$

The image is three times larger than the object.

c $h_i=M\times h_o=3\times 5=15$ mm

23 a A concave lens always forms virtual images.

b $M=\frac{v}{u}=\frac{1}{3}$

$\therefore v=\frac{1}{3}u=\frac{1}{3}\times 15=5$ cm

c Note: Because it is a virtual image, *v* has a negative value.

$\frac{1}{f}=\frac{1}{u}+\frac{1}{v}=\frac{1}{15}+\frac{1}{-5}=\frac{1}{15}-\frac{3}{15}=-\frac{2}{15}$

$\therefore f=-\frac{15}{2}=-7.5$ cm

The focal length of the lens is 7.5 cm. (The negative sign confirms that it is a concave lens.)

24 a Since concave lenses always produce diminished, virtual images, the lens should be convex.

b $M=\frac{v}{u}=30$

$\therefore v=30u=30\times 4.0=120$ cm

$\frac{1}{f}=\frac{1}{u}+\frac{1}{v}=\frac{1}{4}+\frac{1}{120}=\frac{30}{120}+\frac{1}{120}=\frac{31}{120}$

$\therefore f=\frac{120}{31}=3.9$ cm

25 Light can be described using many models. The ray model can be used to predict the behaviour of light in situations involving reflection and refraction.

Light travels in a straight line through a medium of constant density. When a laser is shone through the air the light travels in a straight line. When the light goes from the air into the glass, it bends because of the change in optical density (refractive index). It bends again when it enters the water, and at every other change in optical density.

The water beads have nearly the same optical density as water. When the light is travelling through the water beads in water, it does not bend as there is no change in optical density. When the water beads are in air there is a change in optical density, causing the light to bend entering and exiting each water bead.

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Chapter 11 Thermodynamics

11.1 Heat and temperature

Worked example: Try yourself 11.1.1

CALCULATING THE CHANGE IN INTERNAL ENERGY

A student places a heating element and a paddle-wheel apparatus in an insulated container of water. She calculates that the heater transfers 2530 J of thermal energy to the water and the paddle does 240 J of work on the water.
Calculate the change in internal energy of the water.

Thinking	Working
Heat is added to the system, so Q is positive. Work is done on the system, so W is negative.	$\Delta U = Q - W$ $= 2530 - (-240)$
Note that the units are joules, so express the final answer in J.	$\Delta U = 2770\text{ J}$

11.1 KEY QUESTIONS

1 C. The kinetic particle theory states that the particles in all substances (regardless of their state) are in constant motion.

2 **a** C and D. Initially the chicken is cold and the air in the oven is hot, so the two objects are not in thermal equilibrium. Energy flows from the hot air in the oven to the cold chicken.

b B. As both objects are at same temperature, no transfer of thermal energy takes place. The chicken and the air in the oven are in thermal equilibrium.

3 C and D. Negative kelvin values and Celsius values below –273°C are not possible because temperatures below absolute zero are not possible.

4 The temperature of the gas is just above absolute zero so the particles have very little energy.

5 **a** kelvin = °C + 273 = 30 + 273 = 303 K

b °C = K – 273 = 375 – 273 = 102°C

6 Absolute zero, 10 K, –180°C, 100 K, freezing point of water.

7 $\Delta U = Q - W = -20 - (+50) = -70\text{ kJ}$

11.2 Specific heat capacity

Worked example: Try yourself 11.2.1

CALCULATIONS USING SPECIFIC HEAT CAPACITY

A bath contains 75 L of water. Initially the water is at 50°C.
Calculate the amount of energy that must be transferred from the water to cool the bath to 30°C.

Thinking	Working
Calculate the mass of water. 1 L of water = 1 kg	Volume = 75 L ∴ mass = 75 kg
ΔT = final temperature – initial temperature	$\Delta T = 30 - 50 = -20°\text{C}$
From Table 11.2.1 the specific heat capacity of water is $c_{water} = 4200\text{ J kg}^{-1}\text{ K}^{-1}$. Use the equation $Q = mc\Delta T$.	$Q = mc\Delta T$ $= 75 \times 4200 \times (-20)$ $= -6\,300\,000\text{ J}$ 6.3 MJ must be transferred from the water.

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Worked example: Try yourself 11.2.2

COMPARING SPECIFIC HEAT CAPACITIES

What is the ratio of the specific heat capacity of liquid water to that of steam?

Thinking	Working
Refer to Table 11.2.1 for the specific heat capacities of water in different states.	$c_{water} = 4200\,J\,kg^{-1}\,K^{-1}$ $c_{steam} = 2000\,J\,kg^{-1}\,K^{-1}$
Divide the specific heat of water by the specific heat of steam.	Ratio = $\frac{c_{water}}{c_{steam}} = \frac{4200}{2000}$
Note that ratios have no units since the unit of each quantity is the same and cancels out.	Ratio = 2.1

11.2 KEY QUESTIONS

1 Water requires more energy per degree Celsius to heat it because the specific heat capacity of water is much greater than that of aluminium.

2 If all other variables are the same, aluminium will contain the most thermal energy since it has the highest value for specific heat capacity.

3 100 mL of water has a mass of 0.1 kg.
$Q = mc\Delta T$
$= 0.1 \times 4200 \times (20 - 15)$
$= 2100\,J$

4 150 mL of water has a mass of 0.15 kg.
$Q = mc\Delta T$
$= 0.15 \times 4200 \times (50 - 10)$
$= 25\,200\,J$ or $25.2\,kJ$

5 Remember that both ΔT and mass are proportional to energy.
Use the relationship $Q = mc\Delta T$.
If $x = 10c$ then $20c = 2x$ joules

6 The ratio of the temperature rise is equal to the inverse ratio of the specific heat capacities, that is $\Delta T = \frac{1}{c}$.
Ratio of temperature rise is = $\frac{4200}{900} = 4.67$
$\therefore$ the temperature of the aluminium is 4.67 times that of the water.

7 B. Different states will have different specific heat capacities.

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11.3 Latent heat

Worked example: Try yourself 11.3.1

LATENT HEAT OF FUSION

How much energy must be removed from 5.5 kg of liquid lead at 327°C to produce a block of solid lead at 327°C? Express your answer in kJ.

Thinking	Working
Cooling from liquid to solid involves the latent heat of fusion, where the energy is removed from the lead. Use Table 11.3.1 to find the latent heat of fusion for lead.	$L_{fusion} = 0.25 \times 10^5\,J\,kg^{-1}$
Use the equation: $Q = mL_{fusion}$	$Q = mL_{fusion}$ $= 5.5 \times 0.25 \times 10^5$ $= 1.38 \times 10^5\,J$
Convert to kJ.	$Q = 1.38 \times 10^2\,kJ$

Worked example: Try yourself 11.3.2

CHANGE IN TEMPERATURE AND STATE

3 L of water is heated from a fridge temperature of 4°C to its boiling point at 100°C. It is boiled at this temperature until it is completely evaporated. How much energy in total was required to raise the temperature and boil the water?

Thinking	Working
Calculate the mass of water involved.	3 L of water = 3 kg
Find the specific heat capacity of water.	$c = 4200\,J\,kg^{-1}\,K^{-1}$
Use the equation $Q = mc\Delta T$ to calculate the heat energy required to change the temperature of water from 4°C to 100°C.	$Q = mc\Delta T$ $= 3 \times 4200 \times (100 - 4)$ $= 1\,209\,600\,J$
Find the specific latent heat of vaporisation of water.	$L_{vapour} = 22.5 \times 10^5\,J\,kg^{-1}$
Use the equation $Q = mL$ to calculate the latent heat required to boil water.	$Q = mL$ $= 3 \times 22.5 \times 10^5$ $= 6\,750\,000\,J$
Find the total energy required to raise the temperature and change the state of the water.	Total $Q = 1\,209\,600 + 6\,750\,000$ $= 7\,959\,600\,J$ (or $7.96 \times 10^6\,J$)

11.3 KEY QUESTIONS

1 **a** The mercury is changing state from solid to liquid. It is melting; temperature does not change during phase transitions as average kinetic energy does not change.

b –39°C

c 357°C

d $Q = mL$

$126 = 0.01 \times L$

$L = 126 \div 0.01 = 12\,600$

$= 1.26 \times 10^4\,J\,kg^{-1}$

e $Q = mL$

$3520 - 670 = 0.01 \times L$

$L = 2850 \div 0.01 = 285\,000$

$= 2.85 \times 10^5\,J\,kg^{-1}$.

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2 $Q = mL$
$= 0.1 \times 22.5 \times 10^5$
$= 2.25 \times 10^5\,\text{J}$

3 D. A volatile liquid evaporates rapidly and loses its high-energy molecules. This results in a lower average kinetic energy of the remaining particles and so it cools down.

4 $Q = mL_{\text{fusion}}$
$= 0.2 \times 3.34 \times 10^5$
$= 6.7 \times 10^4\,\text{J}$

11.4 Conduction

Worked example: Try yourself 11.4.1

CALCULATING THE ENERGY TRANSFER BY CONDUCTION THROUGH A BRICK WALL

Calculate the rate of energy transfer by conduction through a 1 m × 1 m square section of a brick wall. Assume the wall is 5 cm thick, the inside temperature is 21°C, and the outside temperature is –4°C. Use Table 11.4.1 on page 318 to find the thermal conductivity of brick.

Thinking	Working
Write out the equation for the rate of energy transfer by conduction.	$\frac{Q}{t} = \frac{kA\Delta T}{d}$
Determine the quantities for the variables *k*, *A*, Δ*T* and *d*.	$k = 1\,\text{W m}^{-1}\,\text{K}^{-1}$ $A = 1\,\text{m}^2$ $\Delta T = 21 - (-4) = 25\,\text{K}$ $d = 0.05\,\text{m}$
Calculate the rate of energy transfer.	$\frac{Q}{t} = \frac{kA\Delta T}{d}$ $= \frac{1 \times 1 \times 25}{0.05}$ $= 500\,\text{W}$

11.4 KEY QUESTIONS

1 The process is quite slow since the mass of the particles is relatively large and the vibrational velocities are fairly low.

2 Metals conduct heat by free-moving electrons as well as by molecular collisions. Wood does not have any free-moving electrons, so it is a poor conductor of heat.

3 Thickness, surface area, nature of the material and the temperature difference between it and the second material.

4 Copper is a better conductor of heat than stainless steel.

5 C. Air is a poor conductor of heat so it limits the transfer of heat.

6 The insulation batts stop the thermal energy from *escaping* the house. The air in the batts has *low* conductivity and the thermal energy is *not able* to escape from the house.

7 Plastic and rubber have low conductivity, so they do not allow the transfer of heat from your hand very easily. Metal has high conductivity, so heat transfers from your hand easily and your hand feels cold.

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11.5 Convection

11.5 KEY QUESTIONS

1 Liquids and gases.
2 Upwards. The heat will rise, while the cooler air/liquid will fall.
3 Air over certain places, such as roads, heats up and as a result becomes less dense. The less dense air rises to form a column of rising air called a thermal.
4 D. Convection is a fast way of transferring thermal energy.
5 It is not possible for solids to transmit heat by convection because solids do not contain the free molecules that are required to establish convection currents.
6 The source of heat, the Sun, is above the water. It takes much longer to heat a liquid when the source is at the top as the convection currents also remain near the top. The warm water is less dense than the cool water and does not allow convection currents to form throughout the water.

11.6 Radiation

11.6 KEY QUESTIONS

1 **a** The light can be partially reflected, partially transmitted and partially absorbed.
b Absorption of light is associated with temperature increase.
2 The higher the temperature of the object, the *higher* the frequency and the *shorter* the wavelength of the radiation emitted. For example, if a particular object emits radiation in the visible range, a cooler one could emit light in the *infrared* range of the electromagnetic spectrum.
3 E. The rate of emission or absorption will depend upon:
- the temperature of the object and of the surrounding environment
- the surface area of the object
- the wavelength of the radiation
- the colour of the object's surface
- the surface characteristics of the object.

4 Conduction and convection require the presence of particles to transfer heat. Heat transfer by radiation can occur in a vacuum as the movement of particles is not required.
5 **a** The matt black beaker cools faster than the others. Matt black objects emit radiant energy faster than shiny, white surfaces.
b The gloss white surface will cool the slowest due to its light colour and shiny finish.
6 Heat sinks are made of dark coloured metals that radiate heat energy strongly and keep the computer cool.

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CHAPTER 11 REVIEW

1 A. The kinetic theory states that particles are in constant motion.

2 Temperature: that is, the average kinetic energy of particles in a substance.

3 Heat refers to the energy that is transferred between objects, whereas temperature is a measure of the average kinetic energy of the particles within a substance.

4 **a** $5 + 273 = 278\,\text{K}$

b $200 - 273 = -73°\text{C}$

5 $\Delta U = Q - W$

$= 75 - (-150)$

$= 225\,\text{J}$

6 $\Delta U = Q - W$

$250 = -300 - W$

$\therefore W = -550\,\text{J}$

The scientist does 550 J of work on the sodium.

7 300 K is 27°C. Higher temperatures mean molecules have greater average kinetic energy. So the average kinetic energy of the hydrogen particles in tank B is greater than the average kinetic energy of the hydrogen particles in tank A.

8 As thermal equilibrium is reached, the balls must be at the same temperature.

9 If 4.0 kJ of energy is required to raise the temperature of 1.0 kg of paraffin by 2.0°C, then 2.0 kJ of energy is required to raise the temperature of 1.0 kg of paraffin by 1.0°C.

So to raise the temperature by 5.0°C, you will need 5 times as much energy, i.e. $5 \times 2.0 = 10.0\,\text{kJ}$ to raise the temperature of 1.0 kg of paraffin by 5.0°C.

Mathematically: Calculate c for paraffin, so $c = \frac{Q}{m\Delta T} = \frac{4000}{1 \times 2} = 2000\,\text{J kg}^{-1}\,\text{K}^{-1}$.

Then for 5.0°C:

$Q = mc\Delta T = 1 \times 2000 \times 5 = 10\,000 = 10\,\text{kJ}$

10 $Q = mc\Delta T$

$10\,500 = 0.25 \times 4200 \times (T - 20)$

$10 = T - 20$

$T = 30$

Final temperature = 30°C

11 $Q = mc\Delta T$

$-13\,200 = m \times 440 \times -30$

$m = -13\,200 \div -13\,200$

$= 1\,\text{kg}$

12 B. Specific heat capacity.

13 This situation describes a change of state, in this case, melting. It occurs because the heat energy is used to increase the potential energy of the particles in the solid instead of increasing their kinetic energy. The energy needed to change from solid to liquid is the latent heat of fusion.

14 Both have the same kinetic energy as their temperatures are the same; however the steam has more potential energy due to its change in state. Therefore the steam has greater internal energy.

15 The higher-energy particles are escaping, leaving behind the lower-energy particles. The result is that the average kinetic energy of the remaining particles decreases, thus the temperature drops.

16 $Q = mc\Delta T$

$c = \frac{Q}{m\Delta T} = \frac{5020}{2.00 \times 20} = 125.5\,\text{J kg}^{-1}\,\text{K}^{-1}$

$= 126\,\text{J kg}^{-1}\,\text{K}^{-1}$.

17 $Q = mL$

$= 0.08 \times 0.88 \times 10^5$

$= 7.0\,\text{kJ}$

18 $\Delta U = Q - W$

$= 14\,600 - (-2.65 \times 10^6)$

$= 2\,664\,600\,\text{J}$

$Q = mL_{\text{fusion}} + mc\Delta T$

$2\,664\,600 = 4.55 \times 3.34 \times 10^5 + 4.55 \times 4200 \times (T - 0)$

$T = 60°\text{C}$

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19 Energy needed to raise the ice from –4.00°C to 0°C:

$Q = mc\Delta T$
$= 0.100 \times 2100 \times 4.00$
$= 840\,\text{J}$

Energy needed to melt the ice at 0°C:

$Q = mL_{\text{fusion}}$
$= 0.100 \times 3.34 \times 10^5$
$= 3.34 \times 10^4\,\text{J}$

Total energy $= 840 + 3.34 \times 10^4$
$= 3.42 \times 10^4$
$= 34\,\text{kJ}$

20 B. Hot water has more fast-moving molecules that are able to break through the surface of the water (evaporate).

21 Polystyrene is a good insulator, whereas metal is a good conductor of heat. When the ice-cube is in thermal contact with the polystyrene, not much heat energy flows from the polystyrene to the ice. The ice-cube melts very slowly as a result. When the ice-cube is in thermal contact with the metal, heat energy flows very quickly from the metal into the ice causing the ice-cube to melt rapidly (recall that heat is always transferred from the hotter to the colder object). When you touch the metal, for the same reason, heat energy flows very rapidly from your skin into the metal causing your fingers to feel cold. As you touch the polystyrene, not much heat energy flows from your fingers so they do not feel as cold.

22 A lot of air is trapped in the down. As air is a poor conductor of heat, the down-filled quilt limits the transfer of heat away from the person.

23 Sweat is a thin layer of water and as it evaporates, it cools down. This is because the higher energy water molecules escape into the air, leaving the lower energy ones behind. The sweat on your body has a large surface area that increases the rate of evaporation. The breeze also increases the rate of evaporation. The cooler layer of sweat remaining on your skin draws heat energy from your skin and so your body cools down.

24 The stopper reduces heat loss by convection and conduction. The air between the walls reduces heat transfer by conduction. The space between the walls is almost a vacuum, so convection currents will not form and heat will be transmitted very slowly, if at all, between the walls. The flask's shiny surface reduces heat transfer by radiation.

25 The person, due to their temperature, emits stronger infra-red radiation than their surroundings. The infra-red radiation is detected by the thermal imaging technology, which, since humans are usually a different temperature from their surroundings, allows them to be 'seen'. The human eye cannot always distinguish a person from their surroundings, especially if they are under cover or if their clothes blend with their surroundings.

26 Water is a better conductor of heat than air. When a person is in cold water, the rate of energy flowing from their body is far greater (25 times) than when in cold air. In cold water, heat energy quickly flows from the warm body into the cold water decreasing the body's temperature to dangerous levels. A wetsuit provides a layer of insulating material (neoprene) around the body and slows the rate of heat loss from the warm body into the cold water. This prevents the person from getting hypothermia.

27 Responses will vary. When thermal energy, or heat, is added to a closed system there are many different things that can happen. The activity looks at how thermal energy increases the temperature.

As the thermal energy (or shaking) adds energy to the system, the molecules in that system travel faster, increasing the average kinetic energy. Temperature is a measure of the average kinetic energy of the molecules, meaning that adding heat to a system can increase the temperature.

This is true in many cases, however if the molecules are moving so fast that it causes the bonds to break, this represents a change of state, although this was not modelled in the activity.

The conduction of heat can also be considered using this model, with the heat being transferred through a series of collisions along the material. Heat transfer through convection can also be related to this model, understanding that increasing the temperature increases the space between the molecules in a fluid as the molecules move faster. This change in density causes natural convection to occur.

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Module 3 Review answers

Waves and thermodynamics

MULTIPLE CHOICE

1 C. Because heat flows from A to B, A is at the higher temperature.
Likewise, C is at a higher temperature than B.
No heat flows from A to C, so they are at the same temperature.

2 B. Temperature is a measure of the average kinetic energy of particles within a system.

3 B.
$\Delta T_{hot} = \Delta T_{cold}$
$T_{final} - 80 = 10 + T_{final}$
$T_{final} = 45°C$

4 C. The larger latent heat required to melt ice to water ($3.34 \times 10^5\,J\,kg^{-1}$ versus $1000\,J\,kg^{-1}\,K^{-1}$ for the specific heat capacity for air) means that more energy is required.

5 C.
K = °C + 273
T (K) = 1550 + 273 = 1823 K

6 B and C. The temperature is related to the average kinetic energy of the particles. This has not changed.
The potential energy has changed as the particles have moved further apart—energy absorbed to overcome the attraction of the particles for one another. Potential energy is increased.
The total internal energy is the sum of potential and kinetic energies. It has increased because potential energy has increased.
Heating does not change the number of particles, just their kinetic energy.

7 A.
$$\begin{aligned} Q &= mL_{fusion} \\ &= 0.150 \times 0.88 \times 10^5 \\ &= 0.132 \times 10^5 \\ &= 13 \times 10^3 \\ &= 13\,kJ \end{aligned}$$

8 A.
$$\frac{I_2}{I_1} = \frac{r_1^2}{r_2^2} = \frac{150^2}{228^2} = \frac{22500}{51984} = 0.43$$
$I_{Mars} = 0.43\,I_{Earth}$

9 D. All waves transfer energy. Mechanical waves need a medium to travel in, but electromagnetic waves do not. Sound waves are an example of a mechanical wave. Visible light is an example of an electromagnetic wave.

10 C and D. The amount of diffraction $\propto \frac{\lambda}{W}$.
Since $v = f\lambda$, $\lambda = \frac{v}{f}$, so the amount of diffraction $\propto \frac{v}{fw}$.
Since f and w are on the bottom, if they are decreased, the amount of diffraction will increase.

11 A, B, C and D. All of these can be explained by treating light as a wave.

12 A and C. The speed of a wave in a string is proportional to the tension and inversely proportional to the mass per unit length. Therefore, the strings with higher tension and lower mass will create a faster wave.

13 B.
$\sin i_c = \frac{1}{n_x}$
$i_c = \sin^{-1}\frac{1}{1.5} = 41.8°$

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14 C. The amplitude can change if energy is lost during the reflection. Frequency, wavelength and period can change only when a wave passes into another medium.

15 C. The angles must be between the ray and the normal.

16 D. $f = \frac{nv}{2l} = \frac{140}{2 \times 0.3} = 233.3\,\text{Hz}$

17 C. Convex mirrors always produce an upright, diminished, virtual image.

18 B.

$I \propto \frac{1}{r^2}$

$I_1 = \frac{1}{r_1^{\,2}}$

$I_2 = \frac{1}{r_2^{\,2}} = \frac{1}{(3r_1)^2} = \frac{1}{9r_1^{\,2}} = \frac{1}{9} I_1$

19 C.

$\lambda = \frac{v}{f}$

$= \frac{340}{280} = 1.21\,\text{m}$

$k = \frac{2\pi}{\lambda} = \frac{2\pi}{1.21} = 5.19 = 5.2\,\text{m}^{-1}$

20 B. The driving frequency needs to equal the natural (or resonant) frequency in order to produce resonance.

SHORT ANSWER

21 Temperature is related to the average kinetic energy of the particles. On sublimation, average kinetic energy of the particles is not altered. Potential energy increases as the molecules move further apart.

22 Energy transferred to water: $Q = mc\Delta T = 4200 \times 0.20 \times (25 - 20) = 4200\,\text{J}$

Energy transferred from metal: $Q = 4200\,\text{J} = c \times 0.10 \times (75 - 25)$

Specific heat capacity of unknown metal: $c = \frac{4200}{0.10 \times 50} = 840\,\text{J}\,\text{kg}^{-1}\,\text{K}^{-1}$

From the table of specific heat capacities this would most closely match aluminium.

23 When heat is absorbed by a material and no phase change is involved, the heat capacity is the energy in joules to heat 1 kg of material by 1°C. For phase changes (latent heat) there is no temperature change; it is merely the energy per kilogram required to cause the phase change.

24 Amplitude = 2 (unknown units)

Period = 4 ms = 0.004 s

Frequency $= \frac{1}{0.004} = 250\,\text{Hz}$

25 A mirror will create a virtual image which can be projected onto a screen (i.e. a piece of card). A virtual image exists on the same side of the mirror as the object being reflected. If the object is sufficiently distant, you could place the piece of card at the focal length of the mirror and the image will be projected onto it.

26 $v_{\text{source}} = 126\,\text{km}\,\text{h}^{-1} = 126 \div 3.6 = 35.0\,\text{m}\,\text{s}^{-1}$

$f' = f\frac{(v_{\text{wave}} + v_{\text{observer}})}{(v_{\text{wave}} - v_{\text{source}})} = 500 \times \frac{340 + 0}{340 - 35}$

$= 500 \times 1.115$

$= 557\ \text{Hz}$

27 $f_{\text{beat}} = |f_1 - f_2|$

$3 = |440 - f_2|$

$\therefore 3 = |440 - 443|$ or $3 = |440 - 437|$

Justine is playing at either 443 Hz or 437 Hz.

To be in tune with Zanni, Justine needs to adjust the tension in her string until the beat disappears. At this point the violins will both be tuned to 440 Hz.

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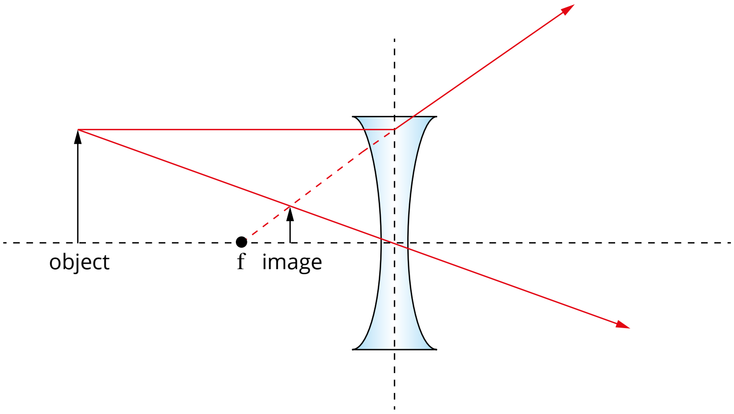
28



29 Pulse 1 reaches the end of the rope at time $t = \frac{7}{1.2} = 5.83\,\text{s}$

At this time pulse 2 has reached a position $s = 1.2 \times (5.83 - 1) = 5.792\,\text{m}$

The two waves are now travelling towards each other at the same speed, so that halfway between $s = 5.8$ and $s = 7$ the waves will interact.

$\therefore \Delta s = (7 - 5.8) \div 2 = 0.602$

$\therefore s = 7 - 0.6 = 6.398 = 6.4\,\text{m}$

30 a $\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$

$\frac{1}{0.15} = \frac{1}{0.50} + \frac{1}{v}$

$v = 0.214 = 21\,\text{cm}$

The image is real.

b $M = -\frac{v}{u} = -\frac{0.214}{0.50} = -0.42$

The image is diminished and inverted.

EXTENDED RESPONSE

31 a Apart from air temperature, it is the heat re-radiated from the ground that affects the night temperatures. Clouds absorb some of the radiated heat and reflect it back to Earth; on clear nights, more energy is lost by radiation.

b The formula for the transfer of energy by conduction (per unit time) is given by:

$$\frac{Q}{t} = \frac{kA\Delta T}{d}$$

You will need to make assumptions about:

- the temperature inside the tent next to the tent wall
- the surface area of the tent
- the thickness of the material
- the thermal conductivity of the material
- the effect of the ground under the tent.

c Responses will depend on the assumptions. For the sample calculation shown here, the following assumptions are made:

- temperature inside the tent next to the tent wall = 7°C
- surface area of tent = $6.0\,\text{m}^2$
- material is 1.0 mm thick
- thermal conductivity of the material (nylon fabric) = $0.13\,\text{W}\,\text{m}^{-1}\,\text{K}^{-1}$
- no heat is lost through the base of the tent.

$\frac{Q}{t} = \frac{kA\Delta T}{d} = \frac{0.13 \times 6.0 \times 10}{0.001} = 7800\,\text{W}$

In 5 hours, $5 \times 7800 = 39\,\text{kWh}$ of energy would be lost.

Even though 7800 J of energy is lost per second in this particular scenario, the hiker will be producing body heat that will radiate into the system. There will also be a temperature gradient in the air inside the tent, so the temperature of the air right next to the tent wall will generally be much less than 7°C.

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32 a The angle of incidence is measured with respect to the normal which is drawn at a right angle to the diamond–air boundary.

$i = 90 - 67.0 = 23.0°$

b $n_1 \sin i = n_2 \sin r$

$2.4 \times \sin 23° = 1.00 \times \sin r$

$\sin r = \frac{n_1 \sin i}{n_2} = \frac{2.4 \times \sin 23°}{1.0} = 0.94$

$r = \sin^{-1} 0.94 = 69.7°$

c $\Delta\theta = r - i$

$= 69.7 - 23.0$

$= 46.7°$

d $v_{\text{diamond}} = \frac{c}{n_{\text{diamond}}}$

$= \frac{3 \times 10^8}{2.4}$

$= 1.25 \times 10^8\,\text{m s}^{-1}$

33 a $Q = mc\Delta T$

$= 0.500 \times 3.8 \times 10^3 \times 25$

$= 47.5\,\text{kJ}$

b Water has a very high specific heat capacity relative to the fats and proteins in the ice cream. Ice cream mix is 70.0% water, hence its lower specific heat.

c The heat lost to the brine at 0°C comes from the latent heat of fusion and the water changes phase and becomes ice.

d Latent heat of fusion for water to freeze:

$Q = mL$

$= 0.7 \times 0.500\text{ kg} \times 3.34 \times 10^5\,\text{J kg}^{-1}$

$\approx 117\,\text{kJ}$

e Total heat gained by brine = heat to cool ice cream mix and freeze water in ice cream

$= 116.9 + 47.5 = 164.4\,\text{kJ}$

$\Delta T = \frac{Q}{mc} = \frac{164.4 \times 10^3\,\text{J}}{5\,\text{kg} \times 3.5 \times 10^3\,\text{J kg}^{-1}\,\text{K}^{-1}}$

Final temperature is 9.39°C above the starting temperature of −11°C.

$T_f \approx -1.6°\text{C}$

Note: After freezing the ice cream will continue to cool until it reaches thermal equilibrium with the brine.

34 a A flute is an open-ended pipe.

$f = \frac{nv}{2l} = \frac{1 \times 340}{2 \times 0.3} = 566.7 = 567\,\text{Hz}$

b $f' = f\frac{(v_{\text{wave}} + v_{\text{observer}})}{(v_{\text{wave}} - v_{\text{source}})} = 566.7\frac{340 + 0}{340 - 5.0} = 575\,\text{Hz}$

This is a very small increase in frequency, so the difference is unlikely to be noticed by the spectator.

c As an object is moving closer while emitting a sound the frequency of the waves becomes compressed so that it has a higher sounding pitch. As the object moves past an observer and the distance starts to increase, the frequency is stretched out so that the pitch decreases.

d $f_{\text{beat}} = |f_1 - f_2|$

$2 = |262 - f_2|$

$\therefore f_2 = 264 \text{ or } 260$

Jeremy is playing at either 264 Hz or 260 Hz.

35 a When the driving frequency is equal to the natural frequency of an object, resonance occurs. This causes a large amount of vibration to occur in the system.

b If resonance isn't taken into account, structures can break over time as the vibrations caused by resonance put the structure under immense pressure.

c $f = \frac{nv}{2l} = \frac{v}{200}$

$\lambda = \frac{2l}{n} = 200\,\text{m}$

d If the driving frequency is equal to the natural frequency of the bridge, resonance occurs and the bridge will begin to vibrate at a large frequency.

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Chapter 12 Electrostatics

12.1 Electric charge

Worked example: Try yourself 12.1.1

THE AMOUNT OF CHARGE ON A GROUP OF ELECTRONS

Calculate the charge, in coulombs, carried by 4 million electrons.

Thinking	Working
Express 4 million in scientific notation.	1 million = 1×10^6 4 million = 4×10^6
Calculate the charge, q, in coulombs by multiplying the number of electrons by the charge on an electron (-1.6×10^{-19} C).	$q = (4 \times 10^6) \times (q_e)$ $= (4 \times 10^6) \times (-1.6 \times 10^{-19}\,C)$ $= -6.4 \times 10^{-13}\,C$

Worked example: Try yourself 12.1.2

THE NUMBER OF ELECTRONS IN A GIVEN AMOUNT OF CHARGE

The net charge on an object is $-4.8\,\mu C$ ($1\,\mu C$ = 1 microcoulomb = 1×10^{-6} C). Calculate the number of extra electrons on the object.

Thinking	Working
Express $-4.8\,\mu C$ in scientific notation.	$q = -4.8\,\mu C$ $= -4.8 \times 10^{-6}\,C$
Find the number of electrons by dividing the charge on the object by the charge on an electron (-1.6×10^{-19} C).	$n_e = \frac{q}{q_e}$ $= \frac{-4.8 \times 10^{-6}\,C}{-1.6 \times 10^{-19}\,C}$ $= 3.0 \times 10^{13}$ electrons

12.1 KEY QUESTIONS

1 They will attract, as they will be oppositely charged.

2 C. The electrostatic force between like charges is repulsive, and between opposite charges attractive.

3 B. An ion. If an atom has gained or lost electrons, it becomes an ion (ionised).

4 $n_e = \frac{q}{q_e} = \frac{-5.0}{-1.6 \times 10^{-19}} = 3.1 \times 10^{19}$ electrons

5 $q = n_e \times q_p = 4.2 \times 10^{19} \times 1.6 \times 10^{-19} = +6.7\,C$

6 Copper is a good conductor of electricity because its electrons are loosely held to their respective nuclei. This allows electrons to move freely through the material by 'jumping' from one atom to the next. Rubber is a good insulator. The rubber coating is used to insulate copper wiring to prevent charge leaving the circuit.

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12.2 Electric fields

Worked example: Try yourself 12.2.1

CALCULATING ELECTRIC FIELD STRENGTH

Calculate the uniform electric field that creates a force of 9.00×10^{-23} N on a proton. ($q_p = +1.602 \times 10^{-19}$ C)

Thinking	Working
Rearrange the relevant equation to make electric field strength the subject.	$\vec{F} = q\vec{E}$ $\vec{E} = \frac{\vec{F}}{q}$
Substitute the values for $\vec{F}$ and q into the rearranged equation and calculate the answer.	$\vec{E} = \frac{\vec{F}}{q}$ $= \frac{9.00 \times 10^{-23}}{1.602 \times 10^{-19}}$ $= 5.62 \times 10^{-4}$ N C^{-1} in the same direction as the force.

Worked example: Try yourself 12.2.2

WORK DONE ON A CHARGE IN A UNIFORM ELECTRIC FIELD

A student sets up a parallel plate arrangement so that one plate is at a potential of 36.0 V and the other earthed plate is 2.00 m away. Calculate the work done to move an electron a distance of 75.0 cm towards the negative plate. ($q_e = -1.602 \times 10^{-19}$ C) In your answer identify what does the work and what the work is done on.

Thinking	Working
Identify the variables presented in the problem to calculate the electric field strength E.	$V_2 = 36.0$ V $V_1 = 0$ V $d = 2.00$ m
Use the equation $\vec{E} = -\frac{V}{d}$ to determine the electric field strength.	$\vec{E} = -\frac{V}{d}$ $\vec{E} = -\frac{0 - 36.0}{2.00}$ $= 18.0$ V m^{-1}
Use the equation $W = qEd$ to determine the work done. Note that d here is the distance that the electron moves.	$W = qEd$ $= -1.602 \times 10^{-19} \times 18.0 \times 0.750$ $= -2.16 \times 10^{-18}$ J
Determine if work is done on the charge by the field or if work is done on the field.	Since the negatively charged electron would normally move away from the negative plate, work is done on the field.

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12.2 KEY QUESTIONS

1 C. In an electric field, a force is exerted between two charged objects.

2 B. The electric field direction is defined as being the direction that a positively charged test charge moves when placed in the electric field.

3 **a** True. Electric field lines start and end at 90° to the surface, with no gap between the lines and the surface.

b False. Field lines can never cross. If they did it would indicate that the field is in two directions at that point, which can never happen.

c False. Electric fields go from positively charged objects to negatively charged objects.

d True. Around small charged spheres called point charges you should draw at least eight field lines: top, bottom, left, right and in between each of these.

e True. Around point charges the field lines radiate like spokes on a wheel.

f False. Between two point charges the direction of the field at any point is the resultant field vector determined by adding the field vectors due to each of the two point charges.

g False. Between two oppositely charged parallel plates the field between the plates is evenly spaced and is drawn straight from the positive plate to the negative plate.

4 $\vec{F} = q\vec{E}$

$= 5.00 \times 10^{-3} \times 2.5$

$= 0.005 \times 2.5$

$= 0.0125$

$= 1.25 \times 10^{-2}\,\text{N}$

5 $\vec{F} = q\vec{E}$

$q = \frac{\vec{F}}{\vec{E}} = \frac{0.025}{18}$

$= 0.00139\,\text{C}$

$= 1.39 \times 10^{-3}\,\text{C}$

$= 1.39\,\text{mC}$

6 $\vec{F} = q\vec{E}$

$= 1.602 \times 10^{-19} \times 3.25$

$= 5.207 \times 10^{-19}\,\text{N}$

$\vec{F} = m\vec{a}$

$\therefore a = \frac{F}{m}$

$= \frac{5.207 \times 10^{-19}}{9.11 \times 10^{-31}}$

$= 5.72 \times 10^{11}\,\text{m}\,\text{s}^{-2}$ (a scalar quantity does not require a direction)

7 $\vec{E} = \frac{V}{d}$

$4000 = \frac{V}{0.3}$

$V = 4000 \times 0.3$

$= 1200\,\text{V}$

8 $\vec{E} = \frac{V}{d}$ has the units $\text{V}\,\text{m}^{-1}$.

Since $\text{V} = \text{J}\,\text{C}^{-1} = \text{kg}\,\text{m}^2\,\text{s}^{-2}\,\text{C}^{-1}$,

$\text{V}\,\text{m}^{-1} = (\text{kg}\,\text{m}^2\,\text{s}^{-2}\,\text{C}^{-1})\,\text{m}^{-1} = \text{kg}\,\text{m}\,\text{s}^{-2}\,\text{C}^{-1}$.

$\vec{E} = \frac{\vec{F}}{q}$ has the units $\text{N}\,\text{C}^{-1}$.

$\text{N} = \text{kg}\,\text{m}\,\text{s}^{-2}$

$\text{N}\,\text{C}^{-1} = \text{kg}\,\text{m}\,\text{s}^{-2}\,\text{C}^{-1}$

$\therefore \text{V}\,\text{m}^{-1} = \text{N}\,\text{C}^{-1}$

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12.3 Coulomb's law

Worked example: Try yourself 12.3.1

USING COULOMB'S LAW TO CALCULATE FORCE

Two small spheres A and B act as point charges separated by 75.0 mm in air. Calculate the force on each point charge if A has a charge of 475 nC and B has a charge of 833 pC. (Use $\varepsilon_0 = 8.8542 \times 10^{-12}\,C^2\,N^{-1}\,m^{-2}$.)

Thinking	Working
Convert all values to SI units.	$q_A = 475 \times 10^{-9} = 4.75 \times 10^{-7}$ C $q_B = 833 \times 10^{-12} = 8.33 \times 10^{-10}$ C $r = 7.50 \times 10^{-}2$ m
State Coulomb's law.	$\vec{F} = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r^2}$
Substitute the values q_A, q_B, r and ε_0 into the equation and calculate the answer.	$\vec{F} = \frac{1}{4\pi \times 8.8542 \times 10^{-12}} \times \frac{4.75 \times 10^{-7} \times 8.33 \times 10^{-10}}{(7.50 \times 10^{-2})^2}$ $\vec{F} = 6.32 \times 10^{-4}$ N repulsion
Assign a direction based on a negative force being attraction and a positive force being repulsion.	$\vec{F} = 6.32 \times 10^{-4}$ N repulsion

Worked example: Try yourself 12.3.2

USING COULOMB'S LAW TO CALCULATE CHARGE

Two small point charges are charged by transferring a number of electrons from q_1 to q_2, and are separated by 12.7 mm in air. The charges on the two points are equal and opposite. Calculate the charge on q_1 and q_2 if there is an attractive force of 22.5 μN between them. (Use $k = 9.0 \times 10^9\,N\,m^2\,C^{-2}$.)

Thinking	Working
Convert all values to SI units.	$\vec{F} = 22.5 \times 10^{-6} = 2.25 \times 10^{-5}$ N $r = 12.7 \times 10^{-3} = 1.27 \times 10^{-2}$ m
State Coulomb's law.	$\vec{F} = k\frac{q_1 q_2}{r^2}$
Substitute the values for $\vec{F}$, r and k into the equation and calculate the answer. (Remember to indicate which charge is positive and which is negative in your final answer.)	$q_1 q_2 = \frac{\vec{F}r^2}{k}$ $= \frac{2.25 \times 10^{-5} \times (1.27 \times 10^{-2})^2}{9.0 \times 10^9}$ $= 4.03 \times 10^{-19}$. Since $q_1 = q_2$: $q_1{}^2 = 4.03 \times 10^{-19}$ $q_1 = \sqrt{4.03 \times 10^{-19}}$ $= +6.35 \times 10^{-10}$ C $q_2 = -6.35 \times 10^{-10}$ C

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Worked example: Try yourself 12.3.3

ELECTRIC FIELD OF A SINGLE POINT CHARGE

Calculate the magnitude and direction of the electric field at point P at a distance of 15 cm to the right of a positive point charge q of 2.0×10^{-6} C.

Thinking	Working
Convert units to SI units as required.	$q = 2.0 \times 10^{-6}$ C $r = 15\,\text{cm} = 0.15\,\text{m}$
Substitute the known values to find the magnitude of E using $E = k\frac{q}{r^2}$.	$E = k\frac{q}{r^2}$ $= 9.0 \times 10^9 \times \frac{2.0 \times 10^{-6}}{0.15^2}$ $= 8.0 \times 10^5$ N C^{-1}
The direction of the field is defined as that acting on a positive test charge. Point P is to the right of the charge.	Since the charge is positive the direction will be away from the charge q, or towards the right.

12.3 KEY QUESTIONS

1 When a positive charge is multiplied by a negative charge the force is negative, and a positive charge attracts a negative charge. When a negative charge is multiplied by a negative charge the force is positive, and a negative charge repels a negative charge.

	Force	q_1 charge	q_2 charge	Action
A	positive	positive	positive	repulsion
B	negative	negative	positive	attraction
B	positive	negative	negative	repulsion
B	negative	positive	negative	attraction

2 D. 24.0×10^3 N C^{-1}. Since the distance has been halved, the inverse square law states that the field is four times the original.

3 $\vec{F} = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r^2}$

$$= \frac{1}{4\pi \times 8.8542 \times 10^{-12}} \times \frac{+1.602 \times 10^{-19} \times -1.602 \times 10^{-19}}{(53 \times 10^{-12})^2}$$

$= -8.21 \times 10^{-8}$ N

4 Recalling that $E = k\frac{q}{r^2}$, then

$E = 9 \times 10^9 \times \frac{3.0 \times 10^{-6}}{(0.05)^2}$

$= 1.1 \times 10^7$ N C^{-1}

5 $\vec{F} = k\frac{q_1 q_2}{r^2}$

$= 9.0 \times 10^9 \times \frac{1.00 \times 1.00}{1000^2}$

$= 9000$ N

6 The magnitudes of the weight force and Coulomb's force must be equal.

Weight force $F = mg = 0.01 \times 9.8 = 0.098$ N

Coulomb's force $F = k\frac{q_1 q_2}{r^2}$

$0.098 = 9 \times 10^9 \times \frac{3.45 \times 10^{-9} \times 6.5 \times 10^{-3}}{r^2}$

$\therefore\ r^2 = 9 \times 10^9 \times \frac{3.45 \times 10^{-9} \times 6.5 \times 10^{-3}}{0.098}$

$= 2.059$

$r = \sqrt{2.059}$

$= 1.435$ m

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7 The force is directly proportional to the product of the charges. The force is inversely proportional to the square of the distance between the point charges.

a If one of the charges is doubled to $+2q$, the force will *double* and *repel*.

b If both charges are doubled to $+2q$, the force will *quadruple* and *repel*.

c If one of the charges is changed to $-2q$, the force will *double* and *attract*.

d If the distance between the charges is halved to $0.5r$, the force will *quadruple* and *repel*.

CHAPTER 12 REVIEW

1 C. Porcelain is a good insulator. It is sometimes used in electric power transmission line distribution, to separate the live power lines from other conductors.

2 $n_e = \frac{q}{q_e}$

$= \frac{-3}{-1.6\times10^{-19}}$

$= 1.9 \times 10^{19}$ electrons

3 $q = n_e \times q_e$

$= 4.2 \times 10^{19} \times 1.6 \times 10^{-19}$

$= 6.7\,\text{C}$

4 $q = n_p \times q_p$

$= 2 \times 1.6 \times 10^{-19}$

$= 3.2 \times 10^{-19}\,\text{C}$

5 B. Silicon. The electrons in a silicon atom are not as tightly bound to the nucleus as electrons in a non-metal.

6 B. Electric field lines start and end at 90° to the surface, with no gap between the lines and the surface. Electric field lines go from positively charged objects to negatively charged objects, and field lines can never cross.

7 $F = qE$

$= 3.00 \times 10^{-3} \times 7.5$

$= 0.003 \times 7.5$

$= 0.0225\,\text{N}$ (a scalar quantity doesn't need a direction)

8 The electrical potential is defined as the work done per unit charge to move a charge from infinity to a point in the electric field. The electrical potential at infinity is defined as zero. When you have two points in an electric field ($\vec{E}$) separated by a distance (d) that is parallel to the field, the potential difference V is then defined as the change in the electrical potential between these two points.

9 $\vec{E} = \frac{V}{d}$

$1000 = \frac{V}{0.025}$

$\therefore V = 1000 \times 0.025 = 25\,\text{V}$

10 C. In a uniform electric field, the electric field strength is the same at all points between the plates.

11 When a positively charged particle moves across a potential difference from a positive plate towards an earthed plate, work is done by the *field* on the *charged particle*.

12 Work done in a uniform electric field is given by $W = qEd$.

$W = qEd$

$= 2.5 \times 10^{-18} \times 556 \times 3.0 \times 10^{-3}$

$= 4.17 \times 10^{-18}\,\text{J}$

13 $\vec{E} = \frac{V}{d}$

$= \frac{15 \times 10^3}{0.12}$

$= 125\,000\,\text{V}\,\text{m}^{-1}$

$\vec{F} = q\vec{E}$

$= 1.6 \times 10^{-19} \times 125\,000$

$= 2 \times 10^{-14}\,\text{N}$ (magnitude of the force is a scalar and doesn't require a direction)

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14 As the oil drop is stationary, the electric force must be equal to the gravitational force. Use the equations $F = mg$ and $F = qE$ to determine the two forces. The number of electrons is found by dividing the charge by the charge of one electron.

$F = mg$
$= 1.161 \times 10^{-14} \times 9.8$
$= 1.138 \times 10^{-13}$ N.

$q = \frac{F}{E}$
$= \frac{1.138 \times 10^{-13}}{3.55 \times 10^{4}}$
$= 3.205 \times 10^{-18}$ C

The number of electrons is found by dividing this value by the charge on one electron:

$n_e = \frac{q}{q_e}$
$= \frac{3.205 \times 10^{-18}}{1.602 \times 10^{-19}}$
$= 20$ electrons

15 **a** work done by the field
b no work is done
c work done on the field
d no work is done
e work done on the field
f work done by the field

16 **a** $W = qEd$
$= 3.204 \times 10^{-19} \times 34 \times 0.01$
$= 1.09 \times 10^{-19}$ J.

b Work is done on the field if the charge is forced to go in a direction it would not naturally go. Alpha particles carry a positive charge. So work is done on the field since a positive charged particle is being moved towards a positive potential.

17 **a** The distance between the charges is doubled to $2r$, so the force will quarter and repel.
b The distance between the charges is halved to $0.5r$, so the force will quadruple and repel.
c The distance between the charges is doubled and one of the charges is changed to $-2q$, so the force will halve and attract.

18 Recall that kinetic energy gained by the ion (K) is equal to work done (W). Therefore, the speed can be calculated using the equation $K = \frac{1}{2}mv^2$ when the kinetic energy is known. Calculate K in two steps by using the work done on a charge in a uniform electric field equation, $W = qEd$, and the equation to determine the electric field, $\vec{E} = -\frac{V}{d}$.

$\vec{E} = \frac{V}{d} = \frac{1000}{0.020} = 50\ 000$ V m^{-1}

$W = qEd = 3 \times 1.602 \times 10^{-19} \times 50\ 000 \times 0.020$
$= 4.806 \times 10^{-16}$ J

$K = \frac{1}{2}mv^2$
$\therefore v = \sqrt{\frac{2K}{m}}$
$= \sqrt{\frac{2 \times 4.806 \times 10^{-16}}{3.27 \times 10^{-25}}}$
$= 5.42 \times 10^{4}$ m s^{-1}

19 $\vec{F} = k\frac{q_1q_2}{r^2}$
$= 9.0 \times 10^{9} \times \frac{5.00 \times 10^{-3} \times 4.00 \times 10^{-9}}{(2.00)^2}$
$= 0.045$ N, repulsive force

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20 The weight force of the plastic ball is given by $F = mg$. The electric charge force is given by $\vec{F} = k\frac{q_1q_2}{r^2}$.

$\therefore mg = k\frac{q_1q_2}{r^2}$

$r^2 = k\frac{q_1q_2}{mg}$

$= 9\times10^9 \times \frac{2.25\times10^{-3}\times 3.05\times10^{-3}}{3\times 9.8}$

$= 2100$

$r = \sqrt{2100}$

$= 45.8\,\text{m}$

21 Find the magnitude of the weight force of the ball using $F = mg$. Then substitute this value into the equation $F = qE$ to calculate the charge.

$F = mg$

$= 5.00\times10^{-3}\times 9.8 = 4.9\times10^{-2}\,\text{N}$

$= qE$

$q = \frac{\vec{F}}{E} = \frac{4.9\times10^{-2}}{300.0}$

$= +1.63\times10^{-4}\,\text{C}$

The charge must be positive to provide an upwards force in the vertically upwards field.

22 There are two protons within the helium nucleus. Recall that a proton has a charge of $q_p = +1.602\times10^{-19}\,\text{C}$. Use Coulomb's law to calculate the force on the protons.

$\vec{F} = k\frac{q_1q_2}{r^2}$

$= 9\times 109\times\frac{1.602\times10^{-19}\times 1.602\times10^{-19}}{(2.5\times10^{-15})^2}$

$= 36.96\,\text{N}$

$\approx 37\,\text{N}$

23 Determine the charge on either point using Coulomb's law:

$\vec{F} = k\frac{q_1q_2}{r^2}$

$1 = 9\times10^9\times\frac{q^2}{(0.30)^2}$

$q = \sqrt{\frac{1\times 0.30^2}{9\times10^9}} = 3.16\times10^{-6}\,\text{C}$

Since each electron has a charge $1.602\times10^{-19}\,\text{C}$, there are $\frac{3.16\times10^{-6}}{1.602\times10^{-19}} = 1.97\times10^{13}$ electrons.

24 Responses will vary.

In electrostatics, like charges (positive and positive or negative and negative) produce a repulsion force between them. In the activity, when two balloons of the same charge are brought near each other they should move apart.

If the charges are opposite, one positive and one negative, an attraction force is created between them. When two oppositely charged balloons are brought near each other they will move together.

When a charged object is brought toward a neutral object, a force is applied to the charges in the material. The protons are held in the nucleus of the atom, but the electrons are able to be moved. This creates an attractive force. If a positively charged balloon is brought close to a neutral surface, the electrons in the surface are attracted toward the balloon, creating a small area of negatively charged surface. The balloon will be attracted to this surface.

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Chapter 13 Electric circuits

13.1 Electric current and circuits

Worked example: Try yourself 13.1.1

QUANTIFYING CURRENT

Calculate the number of electrons that flow past a particular point each second in a copper wire that carries a current of 0.75 A.

Thinking	Working
Rearrange the equation $I = \frac{q}{t}$ to make q the subject.	$I = \frac{q}{t}$ $\therefore I \times t = \left(\frac{q}{t}\right) \times t$ so $q = I \times t$
Calculate the amount of charge that flows past the point in question by substituting the values given.	$q = 0.75 \times 1$ $= 0.75\,C$
Find the number of electrons by dividing the charge by the charge of an electron (1.6×10^{-19} C).	$n_e = \frac{q}{q_e}$ $= \frac{0.75}{1.6\times10^{-19}}$ $= 4.69 \times 10^{18}$ electrons.

13.1 KEY QUESTIONS

1 A continuous conducting loop (closed circuit) must be created from one terminal of a power supply to the other terminal.

2 Cell, light bulb, open switch, resistor, and ammeter.

3 C. In reality charge carriers are electrons, which flow from the negative terminal to the positive terminal of the battery.

4 Use the formula $I = \frac{q}{t}$ in coulombs and seconds.
- **a** 3 A
- **b** 0.5 A
- **c** 0.008 A

5 Use the formula $q = It$ in amperes and seconds.
- **a** 5 C
- **b** 300 C
- **c** 18 000 C

6
- **a** $q = It = (5 \times 10^{-3})(600) = 3\,C$
- **b** $q = 200 \times 5 = 1000\,C$
- **c** $q = (400 \times 10^{-3})(3600) = 1440\,C$

7
- **a** $q = n_e \times q_e$
  $= 1 \times 10^{20} \times 1.6 \times 10^{-19}\,C$
  $= 16\,C$
- **b** $I = \frac{q}{t}$
  $= \frac{16}{4}$
  $= 4\,A$

8
- **a** $n_e = \frac{q}{q_e} = \frac{3.2}{1.6\times10^{-19}} = 2 \times 10^{19}$ electrons
- **b** $I = \frac{q}{t} = \frac{3.2}{10} = 0.32\,A$

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13.2 Energy in electric circuits

Worked example: Try yourself 13.2.1

DEFINITION OF POTENTIAL DIFFERENCE

A car battery can provide 3600 C of charge at 12 V.

How much electrical potential energy (work) is stored in the battery?

Thinking	Working
Recall the definition of potential difference.	$V = \frac{W}{q}$
Transpose the definition of potential difference to make work the subject.	$W = Vq$
Substitute in the appropriate values and solve.	$W = 12 \times 3600$ $= 43\,200\,\text{J}$

Worked example: Try yourself 13.2.2

USING $E = VIt$

A potential difference of 12 V is used to generate a current of 1750 mA to heat water for 7.5 minutes. Calculate the energy transferred to the water in that time.

Thinking	Working
Convert quantities to SI units.	$\frac{1750\text{ mA}}{1000} = 1.75\,\text{A}$ $7.5\text{ min} \times 60\,\text{s} = 450\,\text{s}$
Substitute values into the equation and calculate the amount of energy in joules.	$E = VIt$ $= 12 \times 1.75 \times 450$ $= 9450\,\text{J}$

Worked example: Try yourself 13.2.3

USING P = VI

An appliance running on 120 V draws a current of 6 A. Calculate the power used by this appliance.

Thinking	Working
Identify the relationship needed to solve the problem.	$P = VI$
Identify the required values from the question, substitute and calculate.	$P = 120 \times 6$ $= 720\,\text{W}$

13.2 KEY QUESTIONS

1 A. When a conductor links two bodies between which there is a potential difference, charges will flow through the conductor until the potential difference is equal to zero.

2 **a** $t = 5 \times 60 = 300\,\text{s}$

$E = P \times t$

$= 460 \times 300$

$= 138\,000\,\text{J}$ (or 138 kJ)

b $I = \frac{P}{V} = \frac{460}{230}$

$= 2\,\text{A}$

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3 $V = \frac{W}{q}$

a **i** $\frac{40}{10} = 4\,V$

ii same, 4 V

iii $\frac{20}{10} = 2\,V$

b $I = \frac{q}{t}$

i $\frac{10}{1} = 10\,A$

ii $\frac{10}{10} = 1\,A$

iii $\frac{10}{10} = 1\,A$

4 $V = \frac{W}{q}$

$= \frac{100}{5} = 20\,V$

5 $W = Vq$

$2 \times 10^3 = 12 \times q$

$q = 167\,C$

6 **a** Heat and light.

b $P = \frac{E}{t} = \frac{3600\ J}{60\ s}$

$= 60\,W$

c $I = \frac{P}{V} = \frac{60\ W}{240\ V}$

$= 0.25\,A$

7 The gravitational potential energy of the water.

8 **a** The voltmeter must always be in parallel with the light bulb, M2 or M3.

b The ammeter must always be in series with the light bulb, M1 or M4.

13.3 Resistance

Worked example: Try yourself 13.3.1

USING OHM'S LAW TO CALCULATE RESISTANCE

An electric bar heater draws 10 A of current when connected to a 240 V power supply.
Calculate the resistance of the element in the heater.

Thinking	Working
Ohm's law is used to calculate resistance.	$V = IR$
Rearrange the equation to find R.	$R = \frac{V}{I}$
Substitute in the values for this situation.	$R = \frac{240}{10}$ $= 24\,\Omega$

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Worked example: Try yourself 13.3.2

USING OHM'S LAW TO CALCULATE RESISTANCE, CURRENT AND POTENTIAL DIFFERENCE

The table below shows measurements for the potential difference and corresponding current for an ohmic conductor.

V (V)	0	3	9	V_2
I (A)	0	0.20	I_1	0.80

Determine the missing results, I_1 and V_2.

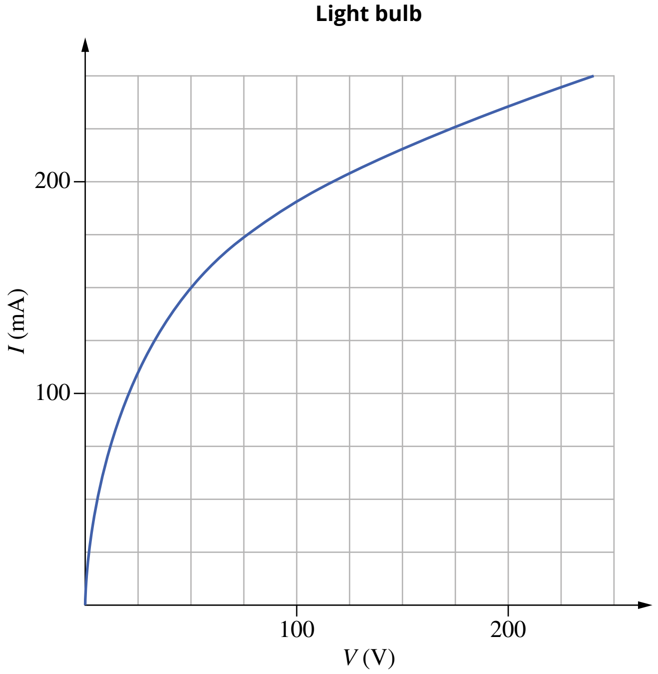
Thinking	Working
Determine the factor by which potential difference has increased from the second column to the third column.	$\frac{9}{3} = 3$ The potential difference has tripled.
Apply the same factor increase to the current in the second column, to determine the current in the third column (I_1).	$I_1 = 3 \times 0.20$ $= 0.6\,\text{A}$
Determine the factor by which current has increased from the second column to the fourth column.	$\frac{0.80}{0.20} = 4$
Apply the same factor increase to the potential difference in the second column, to determine the potential difference in the fourth column (V_2).	$V_2 = 4 \times 3$ $= 12\,\text{V}$

Worked example: Try yourself 13.3.3

CALCULATING RESISTANCE FOR A NON-OHMIC CONDUCTOR

A 240 V, 60 W incandescent light bulb has I–V characteristics shown in the graph.

Calculate the resistance of the light bulb at 175 V.



Thinking	Working
From the graph, determine the current at the required potential difference. Note that current is given in mA, so convert it to A.	At $V = 175\,\text{V}$, $I = 225\,\text{mA}$ $\therefore I = 0.225\,\text{A}$
Substitute these values into Ohm's law to find the resistance.	$R = \frac{V}{I} = \frac{175}{0.225} = 778\,\Omega$

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Worked example: Try yourself 13.3.4

USING OHM'S LAW TO FIND CURRENT

The element of a bar heater has a resistance of 25 Ω. Calculate the current (in mA) that would flow through this element if it is connected to a 240 V supply.

Thinking	Working
Recall Ohm's law.	$V = IR$
Rearrange the equation to make I the subject.	$I = \frac{V}{R}$
Substitute in the values for this problem and solve.	$I = \frac{240}{25} = 9.6\text{ A}$
Convert the answer to the required units.	$I = 9.6\text{ A}$ $= 9.6 \times 10^3\text{ mA}$ $= 9600\text{ mA}$

Worked example: Try yourself 13.3.5

USING OHM'S LAW TO FIND POTENTIAL DIFFERENCE

The globe of a torch has a resistance of 5.7 Ω when it draws 700 mA of current. Calculate the potential difference across the globe.

Thinking	Working
Convert 700 mA to A.	$700 \times 10^{-3} = 0.7\text{ A}$
Recall Ohm's law.	$V = IR$
Substitute in the values for this problem and solve.	$V = 0.7 \times 5.7$ $= 4.0\text{ V}$

13.3 KEY QUESTIONS

1 **a** A, B, C.
b C, B, A. The gradient of each graph is equal to $I \div V$, which is the inverse of the resistance, R. So the steepest gradient actually has the least resistance.

2 $R = \frac{V}{I} = \frac{2}{0.25} = 8\,\Omega$

$I = \frac{V}{R}$

$I_1 = \frac{3}{8} = 0.375\text{ A}$

$V = IR$

$V_2 = 0.60 \times 8 = 4.8\text{ V}$

3 **a** The wire is ohmic. This is because there is a proportional relationship between the voltage and the current, as shown by the linear nature of the I–V graph, which means it obeys Ohm's law.
b 3 A
c $R = \frac{V}{I} = \frac{25}{10} = 2.5\,\Omega$

4 **a** $R = \frac{V}{I} = \frac{2.5}{3.5} = 0.71\,\Omega$
b The resistor is ohmic, as its resistance is constant.

5 They are both right. The resistance of the device is different for different voltages. Therefore the device is non-ohmic.

6 $R = \frac{V}{I} = \frac{5}{45\times10^{-3}} = 111.11\,\Omega$

$I = \frac{V}{R} = \frac{8}{111.11} = 72\text{ mA}$

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7 **a** $R = \frac{\Delta V}{\Delta I} = \frac{4}{2} = 2\,\Omega$

b $I = \frac{V}{R} = \frac{10}{2} = 5\,\text{A}$

8 **a** It is non-ohmic, as the *I*–*V* relationship is non-linear.

b From the graph, when $V = 10\,\text{V}$, $I = 0.5\,\text{A}$

c For $I = 1.0\,\text{A}$, $V = 15\,\text{V}$

d The resistance of the device at these voltages is given by $R = \frac{V}{I}$.

i For $V = 10\,\text{V}$, $R = \frac{10}{0.5} = 20\,\Omega$

ii For $V = 20\,\text{V}$, $R = \frac{20}{1.5} = 13.3\,\Omega$

13.4 Series and parallel circuits

Worked example: Try yourself 13.4.1

CALCULATING AN EQUIVALENT SERIES RESISTANCE

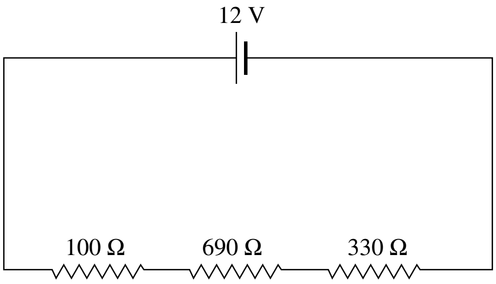
A string of Christmas lights consists of 20 light bulbs connected in series. Each bulb has a resistance of 8 Ω. Calculate the equivalent series resistance of the Christmas lights.

Thinking	Working
Recall the formula for equivalent series resistance.	$R_{series} = R_1 + R_2 + \ldots + R_n$
Substitute in the given values for resistance. As there are 20 equal globes, the problem is easier to answer by multiplying 8 Ω by 20 globes.	$R_{series} = 8 + 8 + 8 + \ldots$ $= 20 \times 8$ $= 160\,\Omega$

Worked example: Try yourself 13.4.2

USING EQUIVALENT SERIES RESISTANCE FOR CIRCUIT ANALYSIS

Use equivalent series resistance to calculate the current flowing in the series circuit below and the potential difference across each resistor.



Thinking	Working
Recall the formula for equivalent series resistance. Find the total resistance In the circuit.	$R_{series} = R_1 + R_2 + R_3 + \ldots + R_n$ $= 100 + 690 + 330 = 1120\,\Omega$
Use Ohm's law to calculate the current in the circuit. Whenever calculating current in a series circuit, use R_T and the voltage of the power supply.	$I = \frac{V}{R} = \frac{12}{1120}$ $= 0.0107\,\text{A}$
Use Ohm's law to calculate the potential difference across each separate resistor.	$V = IR$ $V_1 = 0.0107\,\text{A} \times 100 = 1.07\,\text{V}$ $V_2 = 0.0107\,\text{A} \times 690 = 7.38\,\text{V}$ $V_3 = 0.0107\,\text{A} \times 330 = 3.53\,\text{V}$
Use Kirchhoff's voltage law to check the answer.	$V_T = V_1 + V_2 + V_3$ $= 1.07 + 7.38 + 3.53$ $= 12\,\text{V}$ Since this is the same as the voltage provided by the cell, the answer is reasonable.

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Worked example: Try yourself 13.4.3

CALCULATING AN EQUIVALENT PARALLEL RESISTANCE

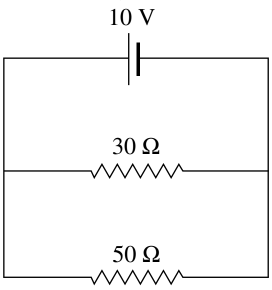
A 20 Ω resistor is connected in parallel with a 50 Ω resistor.
Calculate the equivalent parallel resistance.

Thinking	Working
Recall the formula for equivalent effective resistance.	$-\frac{1}{R_{\text{parallel}}} = \frac{1}{R_1} + \frac{1}{R_2} + \ldots + \frac{1}{R_n}$
Substitute in the given values for resistance.	$\frac{1}{R_{\text{parallel}}} = \frac{1}{20} + \frac{1}{50}$
Solve for R_{parallel}.	$\frac{1}{R_{\text{parallel}}} = \frac{1}{20} + \frac{1}{50} = \frac{5}{100} + \frac{2}{100} = \frac{7}{100}$ $\therefore R_{\text{parallel}} = \frac{100}{7}$ $= 14.3\,\Omega$

Worked example: Try yourself 13.4.4

USING EQUIVALENT PARALLEL RESISTANCE FOR CIRCUIT ANALYSIS

Use an equivalent parallel resistance to calculate the current flowing in the parallel circuit in the circuit diagram below and through each resistor in the circuit.



Thinking	Working
Recall the formula for equivalent parallel resistance.	$\frac{1}{R_{\text{parallel}}} = \frac{1}{R_1} + \frac{1}{R_2} + \ldots - \frac{1}{R_n}$
Substitute the given values for resistance.	$\frac{1}{R_{\text{parallel}}} = \frac{1}{30} + \frac{1}{50}$
Solve for R_{parallel}.	$\frac{1}{R_{\text{parallel}}} = \frac{1}{30} + \frac{1}{50} = \frac{5}{150} + \frac{3}{150} = \frac{8}{150}$ $\therefore R_{\text{parallel}} = \frac{150}{8}$ $= 19\,\Omega$
Use Ohm's law to calculate the current in the circuit. To calculate I, use the voltage of the power supply and the total resistance.	$I_{\text{circuit}} = \frac{V}{R} = \frac{10}{19} = 0.53\,\text{A}$
Use Ohm's law to calculate the current through each resistor. Remember that the voltage through each resistor is the same as the voltage of the power supply, 10 V in this case.	30 Ω resistor: $I_{30} = \frac{V}{R} = \frac{10}{30} = 0.33\,\text{A}$ 50 Ω resistor: $I_{50} = \frac{V}{R} = \frac{10}{50} = 0.20\,\text{A}$
Use Kirchhoff's current law to check the answers.	$I_{\text{circuit}} = I_{30} + I_{50}$ 0.53 A = 0.33 A + 0.20 A This is correct, so the answers are reasonable.

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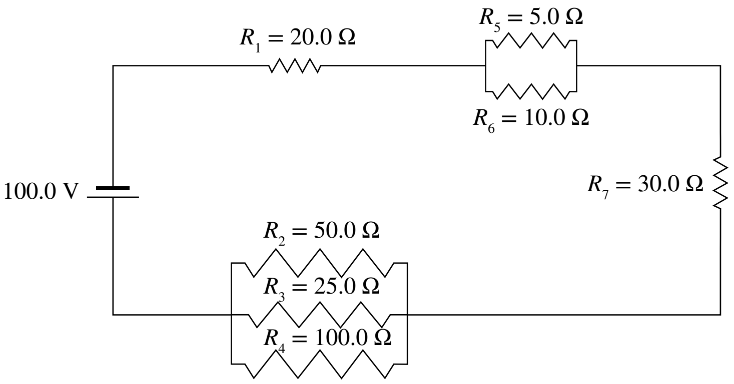
Worked example: Try yourself 13.4.5

COMPLEX CIRCUIT ANALYSIS

Calculate the potential difference across and the current through each resistor in the circuit below.



Thinking	Working	
Find an equivalent parallel resistance for each of the parallel groups of resistors.	For the 10 Ω and 5 Ω resistor group: $\frac{1}{R_{5-6}} = \frac{1}{R_5} + \frac{1}{R_6}$ $= \frac{1}{10} + \frac{1}{5} = \frac{1}{10} + \frac{2}{10} = \frac{3}{10}$ $\therefore R_{5-6} = \frac{10}{3} = 3.33\,\Omega$	Similarly, for the group of three parallel resistors: $\frac{1}{R_{2-4}} = \frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_4}$ $= \frac{1}{25} + \frac{1}{50} + \frac{1}{100}$ $= \frac{4}{100} + \frac{2}{100} + \frac{1}{100} = \frac{7}{100}$ $\therefore R_{2-4} = \frac{100}{7} = 14.3\,\Omega$
Find an equivalent series resistance for the circuit. Note that the circuit can now be thought of as containing four resistors in series.	$R_{series} = 20 + 3.3 + 30 + 14.3 = 67.6\,\Omega$	
Use Ohm's law to calculate the current in the circuit. Use the supply voltage and total resistance to do this calculation.	$I = \frac{V}{R} = \frac{100}{67.6} = 1.48\,A$	
Use Ohm's law to calculate the potential difference across each resistor (or parallel group of resistors) in series.	$V = IR$ $V_1 = 1.48 \times 20.0 = 29.6\,V$ $V_{5-6} = 1.48 \times 3.3 = 4.9\,V$ $V_7 = 1.48 \times 30.0 = 44.4\,V$ $V_{2-4} = 1.48 \times 14.3 = 21.2\,V$ Check: $29.6 + 4.9 + 44.4 + 21.2 \approx 100\,V$ (with slight rounding error)	
Use Ohm's law where necessary to calculate the current through each resistor.	$I_1 = I_7 = 1.48\,A$ $I = \frac{V}{R}$ $I_5 = \frac{4.9}{5} = 0.98\,A$ $I_6 = \frac{4.9}{10} = 0.49\,A$ Check: $0.98 + 0.49 \approx 1.48\,A$ (with slight rounding error). This confirms that Kirchhoff's current law holds for this section.	$I_2 = \frac{21.2}{50} = 0.42\,A$ $I_3 = \frac{21.2}{25} = 0.85\,A$ $I_4 = \frac{21.2}{100} = 0.21\,A$ Check: $0.42 + 0.85 + 0.21$ $= 1.48\,A$ This confirms that Kirchhoff's current law holds for this section.

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Worked example: Try yourself 13.4.6

COMPARING POWER IN SERIES AND PARALLEL CIRCUITS

Consider a 200 Ω and a 800 Ω resistor wired in parallel with a 12 V cell. Calculate the power drawn by these resistors. Compare this to the power drawn by the same two resistors when wired in series.

Thinking	Working
Calculate the equivalent resistance for the parallel circuit.	$\frac{1}{R_{parallel}} = \frac{1}{R_1} + \frac{1}{R_2} + \ldots + \frac{1}{R_n}$ $= \frac{1}{200} + \frac{1}{800} = \frac{4}{800} + \frac{1}{800} = \frac{5}{800}$ $\therefore R_{parallel} = \frac{800}{5} = 160\,\Omega$
Calculate the total current drawn by the parallel circuit.	$V = IR$ $I = \frac{V}{R} = \frac{12}{160} = 0.075\,A$
Use the power equation to calculate the power drawn by the parallel circuit.	$P = VI$ $= 12 \times 0.075 = 0.9\,W$
Calculate the equivalent resistance for the series circuit.	$R_{series} = R_1 + R_2$ $= 200 + 800 = 1000\,\Omega$
Calculate the total current drawn by the series circuit.	$V = IR$ $I = \frac{V}{R_1} = \frac{12}{1000} = 0.012\,A$
Use the power equation to calculate the power drawn by the series circuit.	$P = VI$ $= 12 \times 0.012 = 0.144\,W$
Compare the power drawn by the two circuits.	$\frac{P_{parallel}}{P_{series}} = \frac{0.9}{0.144} = 6.25$ The parallel circuit draws 6.25 times more power than the series circuit.

13.4 KEY QUESTIONS

1 B. The sum of the voltages used by the components must be equal to the supplied voltage.

$R_T = R_1 + R_2$

$= 20 + 20 = 40\,\Omega$

$I_T = \frac{V}{R_{series}} = \frac{6}{40} = 0.15\,A$

V across each resistor:

$V = IR = 0.15 \times 20 = 3\,V$

(Or, as the resistors are equal, the same voltage will be lost across each resistor. This is added to 6 V, so 3 V must be lost across each resistor.)

2 **a** $R_{series} = R_1 + R_2 + R_3$

$= 100 + 250 + 50 = 400\,\Omega$

$I_T = \frac{V}{R_{series}} = \frac{3}{400} = 0.0075 = 7.5\,mA$

b $R = 100\,\Omega$ and $I = 0.0075\,A$

$V_{100} = IR$

$= 0.0075 \times 100 = 0.75\,V$

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3 **a** $\frac{1}{R_{\text{parallel}}} = \frac{1}{R_1} + \frac{1}{R_2}$

$= \frac{1}{20} + \frac{1}{10} = \frac{1}{20} + \frac{2}{20} = \frac{3}{20}$

$R_{\text{parallel}} = \frac{20}{3} = 6.67\,\Omega$

$I = \frac{V}{R}$

$I_T = \frac{5}{6.67}$

$= 0.75\,\text{A}$

b $I_{20} = \frac{V_{20}}{R} = \frac{5}{20} = 0.25\,\text{A}$

c $I_{10} = \frac{V_{10}}{R} = \frac{5}{10} = 0.5\,\text{A}$

4 **a** $V = IR$

$V_{40} = 0.3 \times 40 = 12\,\text{V}$ ($300\,\text{mA} = 0.3\,\text{A}$)

Since the components are in parallel, the voltage across the $40\,\Omega$ resistor (or the $60\,\Omega$ resistor) is also the voltage of the battery.

b $I_{60} = \frac{V_{60}}{R} = \frac{12}{60} = 0.2\,\text{A}$ (or $200\,\text{mA}$)

5 First determine the total resistance of the circuit:

$\frac{1}{R_{3-4}} = \frac{1}{R_3} + \frac{1}{R_4} = \frac{1}{10} + \frac{1}{10}$

$R_{3-4} = 5\,\Omega$

$R_{\text{series}} = 20 + 15 + 5 = 40\,\Omega$

$I_T = \frac{V_T}{R_{\text{series}}} = \frac{12}{40} = 0.3\,\text{A}$ (or $300\,\text{mA}$)

$I_1 = I_2 = I_T = 0.3\,\text{A}$ (since these are in series)

$V_1 = I_1R_1 = 0.3 \times 20 = 6\,\text{V}$

$V_2 = I_2R_2 = 0.3 \times 15 = 4.5\,\text{V}$

$V_3 = V_4 = I_{3-4}R_{3-4} = 0.3 \times 5 = 1.5\,\text{V}$

$I_3 = I_4 = \frac{V_3}{R_3} = \frac{V_4}{R_4} = \frac{1.5}{10} = 0.15\,\text{A}$

6 $R_{\text{top row}} = 3 + 4 = 7\,\Omega$

$R_{\text{bottom row}} = 5 + 6 = 11\,\Omega$

$\frac{1}{R_{\text{parallel}}} = \frac{1}{7} + \frac{1}{11} = \frac{11}{77} + \frac{7}{77} = \frac{18}{77}$

$R_{\text{parallel}} = 4.278 = 4\,\Omega$

7 **a** $R_{\text{series}} = R_1 + R_2 + \ldots + R_n$

$= 20 + 20 + 20 + 20$

$= 80\,\Omega$

$V = IR$

$I = \frac{V}{R} = \frac{10}{80} = 0.125\,\text{A}$

$P = VI$

$= 10 \times 0.125$

$= 1.25\,\text{W}$

b $\frac{1}{R_{\text{parallel}}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_4}$

$= \frac{1}{20} + \frac{1}{20} + \frac{1}{20} + \frac{1}{20} = \frac{4}{20}$

$R_{\text{parallel}} = \frac{20}{4} = 5\,\Omega$

$V = IR$

$I = \frac{V}{R} = \frac{10}{5} = 2\,\text{A}$

$P = VI$

$= 10 \times 2$

$= 20\,\text{W}$

8 C. Parallel wiring allows each appliance to be switched on and off independently (and also receive mains voltage supply).

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CHAPTER 13 REVIEW

1 C. An analogy for electric current must reflect that charges do not leak out of the metal conductor and that since water cannot be compressed, the same amount of water flows in every part of a pipe, just as the electric current is the same in every part of a wire.

2 $I = \frac{q}{t} = \frac{0.23}{60}$

$= 3.8 \times 10^{-3}$ A

3 Conventional current represents the flow of charge around a circuit as if the moving charges were positive, which means the direction is from the positive terminal to the negative terminal. In reality, the moving particles in a metal wire are negatively charged electrons. Electron flow describes the movement of these electrons from the negative terminal to the positive terminal.

4 $\frac{1}{R_{\text{parallel}}} = \frac{1}{R_1} + \frac{1}{R_2} = \frac{1}{R_1} + \frac{1}{R_1} = \frac{2}{R_1}$ (The resistors are identical, so $R_1 = R_2$.)

$\therefore R_{\text{parallel}} = \frac{R_1}{2}$

$R_1 = 2 \times R_{\text{parallel}} = 2 \times 68 = 136\,\Omega$

5 $V = \frac{W}{q} = \frac{2}{0.5} = 4\,\text{V}$

6 C. Using $E = VIt$, you would need to measure current, time and potential difference.

7 $P = \frac{E}{t} = \frac{2500}{30 \times 60} = 1.39\,\text{W}$

8 $I = \frac{P}{V} = \frac{2000}{230} = 8.7\,\text{A}$

Note that the time the oven is left on does not affect the current draw.

9 $R = \frac{V}{I} = \frac{2.5}{5} = 0.5\,\Omega$

10 $R = \frac{V}{I} = \frac{240}{0.25} = 960\,\Omega$

11 $V = IR = 0.25 \times 80 = 20\,\text{V}$

12 $R = \frac{V}{I} = \frac{1.5}{0.05} = 30\,\Omega$

13 **a** At 1 V, $I = 1$ A.

$R = \frac{V}{I} = \frac{1}{1} = 1\,\Omega$

b At 7 V, $I = 3.5$ A.

$R = \frac{V}{I} = \frac{7}{3.5} = 2\,\Omega$

c At 12 V, $I = 4$ A.

$R = \frac{V}{I} = \frac{12}{4} = 3\,\Omega$

14 As electrons travel through a piece of copper wire, they constantly bump into copper ions that slow them down. Resistance is a measure of how hard it is for current to flow through a particular material.

15 $E = VIt$

$= 240 \times 5 \times (3 \times 60)$

$= 216\,000\,\text{J}$ or $216\,\text{kJ}$

16 $P = VI = 240 \times 0.5 = 120\,\text{W}$

17 $I = \frac{P}{V} = \frac{1600}{240} = 6.67\,\text{A}$

$R = \frac{V}{I} = \frac{240}{6.67} = 36\,\Omega$

18 **a** $q = (0.3\,\text{A}) \times (60\,\text{s}) = 18\,\text{C}$

b $E = qV = 18 \times 3 = 54\,\text{J}$

c The energy is provided by the battery.

19 A. The equivalent resistance of series resistors is the sum of their individual resistances.

20 **a** $R_T = R_{\text{parallel pair}} + R_3$

$R_3 = R_T - R_{\text{parallel pair}}$

$= 8.5 - 5 = 3.5\,\Omega$

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b $I_3 = I_T = \frac{V_T}{R_T} = \frac{3}{8.5} = 0.3529 = 0.35\,A$

c $V_3 = I_3 \times R_3 = 0.3529 \times 3.5 = 1.235 = 1.2\,V$

$V_{\text{parallel pair}} = 3 - 1.235 = 1.765 = 1.8\,V$

d $I_2 = \frac{V_T}{R_T} = \frac{1.765}{15} = 0.1176 = 0.12\,A$

e $I_1 = I_T - I_2 = 0.3529 - 0.1176 = 0.2353 = 0.24\,A$

f $R_1 = \frac{V_1}{I_1} = \frac{1.765}{0.2353} = 7.50\,\Omega$

21 **a** Ammeter. The meter is connected in series so it must be an ammeter.

b $\frac{1}{R_{\text{top parallel group}}} = \frac{1}{40} + \frac{1}{40} = \frac{2}{40}$

$R_{\text{top parallel group}} = 20\,\Omega$

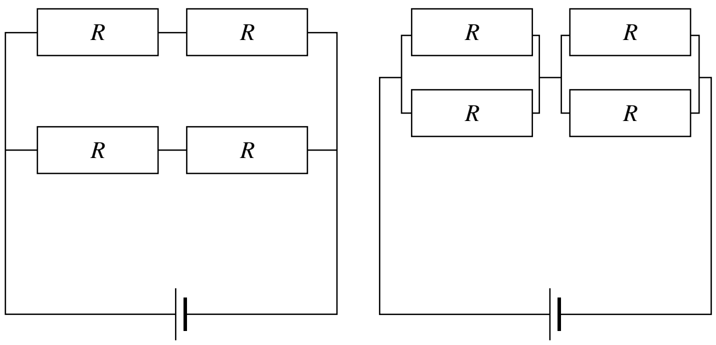
$\frac{1}{R_{\text{bottom parallel group}}} = \frac{1}{20} + \frac{1}{60} = \frac{3}{60} + \frac{1}{60} = \frac{4}{60}$

$R_{\text{bottom parallel group}} = 15\,\Omega.$

$\frac{1}{R_{\text{total}}} = \frac{1}{20} + \frac{1}{15} = \frac{3}{60} + \frac{4}{60} = \frac{7}{60}$

$R_{\text{total}} = \frac{60}{7} = 8.57\,\Omega$

22 The circuit needs either two pairs of series resistors connected in parallel or two pairs of parallel resistors connected in series.



23 $R_{\text{series}} = R_1 + R_2 = 600 + 1200 = 1800\,\Omega$

$I = \frac{V}{R} = \frac{9}{1800} = 0.005\,A$

$V_{600} = IR$

$= 0.005 \times 600 = 3\,V$

24 B.

$V_{\text{out}} = IR_{400}$

$8 = I \times 400$

$I = 0.02\,A$

$V_R = IR_R$

$12 = 0.02 \times R_R$

$R = 600\,\Omega$

Alternatively,

$R : 400\,\Omega = 12\,V : 8\,V$

$\therefore R = 400 \times \frac{12}{8}$

$= 600\,\Omega$

25 **a** $R_{\text{series}} = R_1 + R_2 + R_3 = 20 + 20 + 20 = 60\,\Omega$

$V = IR$

$\therefore I = \frac{V}{R} = \frac{12}{60} = 0.2\,A$

$P = VI = 12 \times 0.2 = 2.4\,W.$

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b $\frac{1}{R_{\text{parallel}}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} = \frac{1}{20} + \frac{1}{20} + \frac{1}{20} = \frac{3}{20}$

$R_T = \frac{20}{3} = 6.67\,\Omega$

$I = \frac{V}{R} = \frac{12}{6.67} = 1.8\,\text{A}$

$P = VI = 12 \times 1.8 = 21.6\,\text{W}$

c $\frac{P_{\text{parallel}}}{P_{\text{series}}} = \frac{21.6}{2.4} = 9$

The parallel circuit draws 9 times more power.

26 Responses will vary. In the electric circuit, current can be described as the charge per unit time: $I = \frac{q}{t}$.

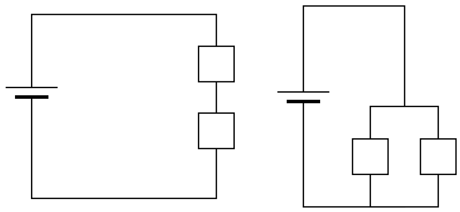
In the water circuit, the flow rate can be described as the volume of water per unit time: flow rate = $\frac{V}{t}$.

The smaller-diameter tubes restrict the flow of water, in an analogous way to resistors in the electric circuit.

The water circuit and electric circuit follow the same relationship for the current in series and in parallel.

The energy in the water circuit is transformed from potential into kinetic energy. In an electric circuit, the chemical potential supplied by a battery is transformed into different types of energy, such as kinetic energy of the electrons.

The two electric circuits are shown in the figure below.



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Chapter 14 Magnetism

14.1 Magnetic materials

14.1 KEY QUESTIONS

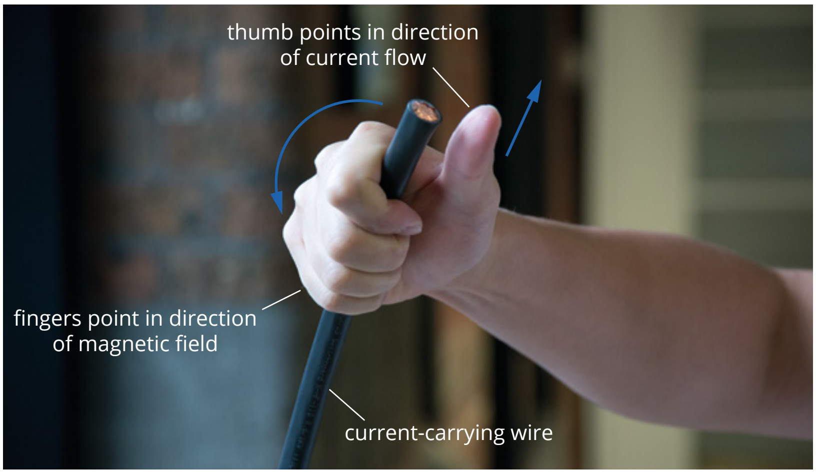
1 B. A north pole is always associated with a south pole. The field around the magnet is known as a dipole field. All magnets are dipolar. This means that every magnet always has a north and a south pole.
2 Iron, cobalt, nickel.
3 Like magnetic poles repel each other; unlike magnetic poles attract each other.
4 The magnetic field strength of the material would increase. Aligning all the magnetic domains in a ferromagnetic material would create a larger magnetic field.
5 B. The bulk piece of a ferromagnetic material is divided into magnetic domains. A magnetic domain is a region in the material where the magnetic field is aligned.
6 When the external magnetic field is removed, the individual magnetic domains remain fixed in their new orientation and the newly aligned domains produce a uniform resultant magnetic field. This is how a ferromagnetic material is magnetised.
7 The magnetic force is a non-contact force. A magnetic material produces a non-contact force (or force mediated by a field) around itself.

14.2 Magnetic fields

Worked example: Try yourself 14.2.1

DIRECTION OF THE MAGNETIC FIELD

A current-carrying wire runs along the length of a table. The conventional current direction, *I*, is running towards an observer standing at the near end.
What is the direction of the magnetic field created by the current as seen by the observer?

Thinking	Working
Recall that the right-hand grip rule indicates the direction of the magnetic field.	Point your thumb to the front in the direction of the current flow. Hold your hand with your fingers aligned as if gripping the wire. 
Describe the direction of the field in relation to the reference object or wire, in a way that can be readily understood by a reasonable reader.	The magnetic field direction is perpendicular to the wire. As the current travels along the wire, the magnetic field runs anticlockwise around the wire.

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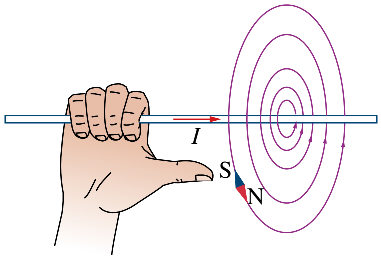
14.2 KEY QUESTIONS

1 C. If you consider the spacing of the magnetic field in the loop as shown by the crosses and dots, it is already non-uniform. Turning the current on and off creates a changing field around the loop but the loop's magnetic field is still non-uniform.

2 The direction of the magnetic field created by the current is perpendicular to the wire and runs up the front of the wire then down the back when looking from the front of the wire.



3 The end labelled A is the north pole. Use the right-hand grip rule to find the field in the conductor.

4 **a** Based on the directions provided, the magnetic field would be towards the east—away from the north pole of the left-hand magnet.
b Based on the directions provided, the magnetic field would be towards the west—away from the north pole of the right-hand magnet.
c A magnetic field is a vector. If a point is equidistant from two magnets and the directions of the two fields are opposite, the vector sum is zero.

5 **a** A = east, B = south, C = west, D = north.
b A = west, B = north, C = east, D = south.

14.3 Calculating magnetic fields

Worked example: Try yourself 14.3.1

MAGNITUDE OF THE MAGNETIC FIELD

A charged wire carries a current of 2 A. What is the magnetic field created by the wire at a distance of 10 cm?

Thinking	Working
Recall the formula used to calculate the magnetic field surrounding a current-carrying wire.	$B = \frac{\mu_0 I}{2\pi r}$
Substitute known values into the equation.	$B = \frac{1.257 \times 10^{-6} \times 2}{2\pi \times 0.1}$
Solve for the magnetic field B.	$B = 4 \times 10^{-6}$ T

Worked example: Try yourself 14.3.2

MAGNITUDE AND DIRECTION OF THE MAGNETIC FIELD

A current-carrying wire runs horizontally across a table. The conventional current direction, I, is running from left to right. The current-carrying wire produces a magnetic field of 3×10^{-6} T measured 15 cm from the wire.

a What current must the wire be carrying to produce that field?	
Thinking	**Working**
Recall the formula used to calculate the magnetic field surrounding a current-carrying wire.	$B = \frac{\mu_0 I}{2\pi r}$
Re-arrange the formula to solve for the unknown quantity.	$I = \frac{2\pi r B}{\mu_0}$
Substitute in known values into the equation.	$I = \frac{2\pi \times 0.15 \times 3 \times 10^{-6}}{1.257 \times 10^{-6}}$
Solve for the current I.	$I = 2.25$ A

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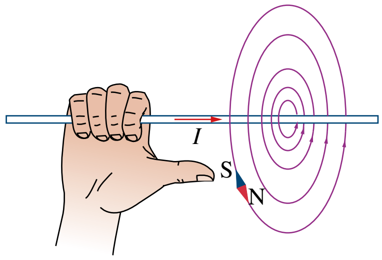
b Draw a diagram showing the direction of the magnetic field around the wire.	
Thinking	**Working**
Use the right-hand rule to determine the direction of the magnetic field.	The direction of the magnetic field created by the current is perpendicular to the wire. It travels down the front of the wire then up the back when looking from the front of the wire.
Draw a diagram of the magnetic field lines.	

Worked example: Try yourself 14.3.3

MAGNITUDE OF THE MAGNETIC FIELD IN A SOLENOID

A solenoid of length 1 m with $N = 100$ turns carries a current of 2.0 A.

a What is the magnetic field created by this current at a point well inside the solenoid?	
Thinking	**Working**
Recall the formula used to calculate the magnetic field in a solenoid.	$B = \frac{\mu_0 N I}{L}$
Determine what values to use, in SI units.	$\mu_0 = 1.257 \times 10^{-6}\,\text{T A}^{-1}$ $I = 2\,\text{A}$ $N = 100$ $L = 1\,\text{m}$
Substitute known values into the equation.	$B = \frac{1.257 \times 10^{-6} \times 100 \times 2}{1}$
Solve for the magnetic field B.	$B = 2.5 \times 10^{-4}\,\text{T}$

b If the number of turns is decreased to 50, what effect would that have on the magnetic field strength?	
Thinking	**Working**
Recall the formula for calculating the magnetic field in a solenoid.	$B = \frac{\mu_0 N I}{L}$
Using the formula to calculate the magnetic field in a solenoid, determine the effect of halving N.	$B_1 = \frac{\mu_0 N_1 I}{L}$ If $N_2 = 50 = 0.5N_1$, then: $B_2 = \frac{\mu_0 \times 0.5 N_1 I}{L}$ $= 0.5 \times B_1$ The magnetic field would be halved.

14.3 KEY QUESTIONS

1 The magnetic field strength is directly proportional to the current in the current-carrying wire. This is defined by Ampere's law.

2 False. The magnetic field at a point well inside a solenoid is uniform and independent of the length or diameter of the solenoid. Instead, it is dependent upon the number of turns of the coil (N) per unit length (L) of the solenoid.

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3 **a** The magnetic field strength would increase. The more tightly wound the wire around a solenoid, the greater the magnetic field.
b The magnetic field strength would also increase by a factor of four: $B = 24 \times 10^{-6}$ T.
c The magnetic field strength would also decrease by a factor of two: $B = 3 \times 10^{-6}$ T.

4 As defined by Ampere's law, the strength of the magnetic field is directly proportional to the current in the current-carrying wire and the number of turns per unit length of the solenoid. If you increase the current or the number of turns per unit length, the magnetic field strength will increase.

5 $B = \frac{\mu_0 I}{2\pi r} = \frac{1.257 \times 10^{-6} \times 1}{2\pi \times 0.02} = 1 \times 10^{-5}$ T

6 $B = \frac{\mu_0 I}{2\pi r}$

$\therefore I = \frac{2\pi r B}{\mu_0} = \frac{2\pi \times 0.1 \times 2 \times 10^{-6}}{1.257 \times 10^{-6}} = 1$ A

7 Recall that the magnetic field strength through a solenoid is governed by Ampere's law:

$B = \frac{\mu_0 N I}{L}$

a The magnetic field strength would double to 12×10^{-4} T.
b The magnetic field strength would be halved to 3×10^{-4} T.
c The magnetic field strength would increase by a factor of 4 to 24×10^{-4} T.
d The magnetic field strength would decrease by a factor of 4 to 1.5×10^{-4} T.

8 Using Ampere's law for a current-carrying wire:

$B = \frac{\mu_0 I}{2\pi r}$

Remember that one newton is equivalent to one kg m s^{-2}. Replacing each variable with the appropriate unit yields:

$B = \frac{(\text{N A}^{-2})(\text{A})}{(\text{m})} = (\text{kg m s}^{-2})(\text{A}^{-1})(\text{m}^{-1}) = \text{kg s}^{-2}\,\text{A}^{-1}$

CHAPTER 14 REVIEW

1 A and D. Opposite poles attract, like poles repel.

2 Magnets always have two poles. If you break a magnet in half, the resulting magnets also have two poles. For this reason, magnets are said to be dipolar. You cannot have a single magnetic pole (south pole or north pole).

3 The bulk piece of a ferromagnetic material is divided into magnetic domains. The magnetic domains in a ferromagnetic material can be aligned using an external magnetic field, resulting in a stronger magnet.

4 Iron is a common material chosen, as it is a ferromagnetic material. Nickel and cobalt could also be used for the same reason. If a current-carrying wire is wrapped around an iron core, it produces a magnetic field. That magnetic field will align the magnetic domains in the iron. Transformers and solenoids use iron as a core for this reason.

5 There are two types of forces: contact and non-contact forces (or forces mediated by a field). A magnetic material produces a non-contact force, whereas the force between two objects that collide is a contact force.

6 The bulk piece of a ferromagnetic material is divided into magnetic domains. A magnetic domain is a region in the material where the magnetic field is aligned. The magnetic fields in separate magnetic domains point in different directions, which causes them to cancel out. If an external magnetic field is applied, the individual magnetic domains align with the external magnetic field, resulting in a magnetised material.

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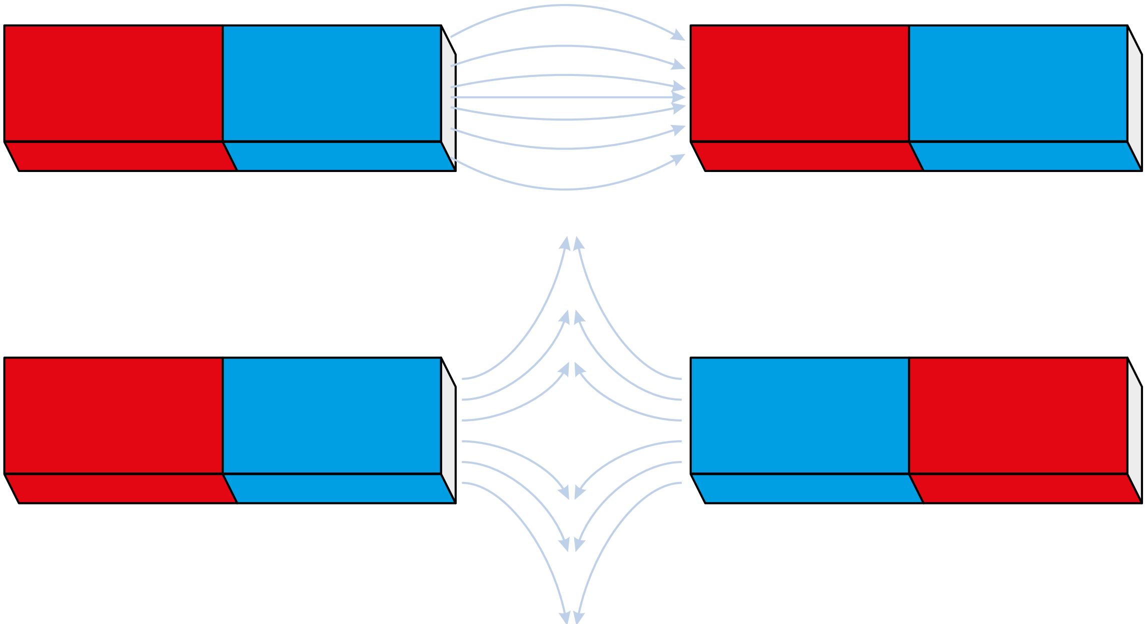
7 The field lines between two like poles diverge between the two poles, as the forces are equal and opposite. The field lines between two opposite poles will go from the north pole of one magnet to the south pole of the other magnet. A diagram of these is shown below.



8 True. As the distance from the magnet decreases, the magnetic field is spread over a smaller area and its strength at any point increases. This is represented by denser (closer) magnetic field lines.

9 The right-hand grip rule.

10 **a** With the current turned off the loop is producing no field. The steady field in the region would be the only contributing field. It has a value of B into the page.

b With the current doubled, the loop is producing double the field, $2B$. The steady field in the region would be contributing B. The total is $3B$ into the page.

c The field from the loop would exactly match that of the field in the region but in the opposite direction. The vector total would be zero.

11 Two current-carrying wires arranged parallel to each other each have their own magnetic field. The direction of the magnetic field around each wire is given by the right-hand grip rule. If the current in each wire is running the same direction, and the two wires are brought close together, their magnetic fields would be in opposite directions, and the two wires would attract.

12 **a** The direction at point B would be South.

b The direction at point D would be North. There would be no change to the direction based on the increased current, as the direction of the magnetic field is independent of the magnitude of the current.

c The direction at point A would be West. There would be no change to the direction based on the increased current, as the direction of the magnetic field is independent of the magnitude of the current.

13 The magnetic field strength is inversely proportional to the distance of the field measurement from the current-carrying wire. Recall Ampere's law:

$B = \frac{\mu_0 I}{2\pi r}$

If the magnetic field is measured further away from the current-carrying wire, the magnetic field strength would be less.

14 C. The magnetic field at a point well inside a solenoid is uniform and independent of the length or diameter of the solenoid. Instead, it is dependent upon the number of turns of the coil (N) per unit length (L) of the solenoid.

15 **a** Recall Ampere's law for the magnetic field in a solenoid:

$B = \frac{\mu_0 N I}{L}$

If the current is increased by a factor of four (10 A to 40 A), and the number of turns per unit length is reduced by a factor of two, the resulting magnetic field strength would double to 12×10^{-4} T.

b Recall Ampere's law for the magnetic field in a solenoid:

$B = \frac{\mu_0 N I}{L}$

If the current is decreased by a factor of 2 (10 A to 5 A), and the number of turns per unit length is reduced by a factor of 2, the resulting magnetic field strength would be reduced by a factor of 4 to 1.5×10^{-4} T.

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c Recall Ampere's law for the magnetic field in a solenoid:

$B = \frac{\mu_0 NI}{L}$

If the magnetic field remained the same, and the number of turns was increased by a factor of 10, the current required to generate that same magnetic field would also be reduced by a factor of 10 (10 A to 1 A).

16 Recall Ampere's law for the magnetic field in a solenoid:

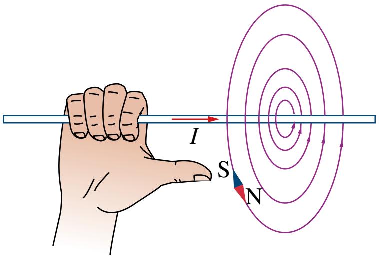
$B = \frac{\mu_0 NI}{L}$

To keep the power consumption low, you would want to reduce the current used as much as possible (recall from Ohm's law that the power lost through the resistance of the wire is $P = I^2 \times R$). Increasing the number of turns in the solenoid increases the magnetic field strength without requiring an increase to the current.

17 **a** $B = \frac{\mu_0 I}{2\pi r}$

$\therefore I = \frac{2\pi rB}{\mu_0} = \frac{2\pi \times 0.08 \times 2.8 \times 10^{-6}}{1.257 \times 10^{-6}} = 1.1 \text{ A}$

b The direction of the magnetic field created by the current is perpendicular to the wire and runs up the front of the wire then down the back when looking from the front of the wire.



18 **a** $B = \frac{\mu_0 I}{2\pi r}$

$\therefore r = \frac{\mu_0 I}{2\pi B} = \frac{1.257 \times 10^{-6} \times 3}{2\pi \times 5.9 \times 10^{-6}} = 0.1 \text{ m}$

b $B = \frac{\mu_0 I}{2\pi r}$

$\therefore r = \frac{\mu_0 I}{2\pi B} = \frac{1.257 \times 10^{-6} \times 10}{2\pi \times 5.9 \times 10^{-6}} = 0.3 \text{ m}$

19 $B = \frac{\mu_0 NI}{L}$

$\therefore I = \frac{BL}{\mu_0 N} = \frac{7.2 \times 10^{-6} \times 0.5}{1.257 \times 10^{-6} \times 100} = 0.0286 \text{ A or } 28.6 \text{ mA}$

20 The locking mechanism is a magnetic lock. When the electromagnet is energised, the magnetic strength increases and causes the lock to close. When the electromagnetic is disabled, or the current in the current-carrying wire is stopped, the lock will open. An automated locking mechanism could be created where an electric circuit either passes current to the electromagnet to close the lock, or the electric circuit is opened to open the lock.

21 Responses will vary. A current-carrying wire produces a magnetic field. In this inquiry activity, the wire is wrapped into a coil or a solenoid, concentrating the magnetic field. The strength of the field is related to the number of coils, the current and the length of the solenoid.

When the electricity is turned on, a magnetic field is produced, and the metal rod experiences a force. Metal objects are attracted to both poles of a magnet. When the rod is placed on the opposite side it is still attracted.

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Module 4 Review answers

Electricity and magnetism

MULTIPLE CHOICE

1 B. A voltmeter will be placed in parallel with the element, while an ammeter will be placed in series to measure the current.

2 A.

3 B. Kirchhoff's current law tells us that all current coming from the first element must pass through the second when they are connected in series. If the two elements are identical, the other alternatives may also be correct, but that is only true in that special case.

4 C. The current is split at junctions, while the voltage in parallel will be the same.

5 A. L_1 will be brighter. With the three LEDs in the circuit there is 1.0 V across each. As the switch shorts out L_3, the 3.0 V is shared between the other two, giving them 1.5 V each and hence making them brighter.

6 D. Treating the wires as having no resistance, the current through L_3 will be zero as it will simply short through the switch path. Therefore, the light at L_3 will turn off.

7 C. The reading on voltmeter V = terminal voltage of battery. But because the battery has zero internal resistance the terminal voltage = EMF of battery, which remains constant regardless of the magnitude of the current.

8 A. Closing S increases the current flowing through the battery and hence the power it is producing.

9 A. In a series circuit the current must be the same in every part of the circuit.

10 D. Connecting another resistor in parallel with R_1 would reduce the effective resistance of the circuit, thereby increasing the current through R_2 and since $V = IR$, the voltage across R_2 would also increase.

11 A. The power output of the battery is given by $P = VI$ and since the current would increase, P would also increase.

12 D. The most important thing is to disconnect the power.

13 B and C. The electrostatic force is a non-contact force experienced by a charged object in an electric field.

14 A, B and C. A will produce a greater total force, B will increase the current, and C will result in a stronger magnetic field through the coil. D would reduce the current.

15 C. $1.32 \times 10^7 = 9\times10^9 \frac{3\times10^{-6}}{r^2}$

$r^2 = \frac{9\times10^9\times3\times10^{-6}}{1.32\times10^7} = 20.45 \times 10^{-4}$

$r = 0.045$ m = 4.5 cm

16 D.

$\vec{F} = q\vec{E}$

$= 1.602 \times 10^{-19} \times 6.7$

$= 10.7 \times 10^{-19}$ N

and

$\vec{F} = m\vec{a}$

$\vec{a} = \frac{\vec{F}}{m}$

$= \frac{10.7\times10^{-19}}{9.11\times10^{-31}}$

$= 11.8 \times 10^{11}$ m s^{-2}

17 A. The other diagrams are either non-uniform (B and C) or not as strong (D).

18 A.

$B = \frac{\mu_0 I}{2\pi r}$

$= \frac{6.8\times10^{-6}\times20\times10^{-3}}{2\pi\times0.011}$

$= 1.97 \times 10^{-6}$ T

$\approx 2 \times 10^{-6}$ T

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19 A. The direction is not an indication of magnitude. Field lines can never cross. Evenly spaced field lines indicates a uniform field.

20 B. Your thumb will be in the direction of the current.

SHORT ANSWER

21 Current only flows when there is a potential difference. If the bird touches only one wire the potential difference between the feet is negligible. If it touches two different wires they will be at different voltages, and so current will flow.

22 a A current of $I = \frac{V}{R} = \frac{5.0}{4.0 \times 10^3} = 1.3\,\text{mA}$ will flow through the $4.0\,\text{k}\Omega$ resistor.

So a current of 0.7 mA needs to flow through the second resistor in parallel with it. The value of this resistor:

$R = \frac{V}{I}$

$= \frac{5.0}{0.7} = 6.7\,\text{k}\Omega$

b $\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2} = \frac{1}{4.0} + \frac{1}{6.7}$

$\therefore R_T = 2.5\ \text{k}\Omega$

or

$R_T = \frac{V}{I} = \frac{5.0}{2 \times 10^{-3}} = 2.5\,\text{k}\Omega$

23 a $P = IV$

$V = \frac{P}{I} = \frac{6.0}{0.50} = 12\,\text{V}$

b 8 V. The voltage supplied by the battery will be used by the components of the system so that $20 = 12 + V_R$

c 16 Ω. Using $V = IR$, and substituting the values $V = 8\,V$ and $I = 0.5\,\text{A}$.

24 Household circuits are connected in parallel, so that each device is supplied with 240 V and can be turned on and off individually. Within a single circuit all the current to the devices in parallel passes through the one circuit breaker.

25 $\vec{F} = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r^2}$

$= \frac{9 \times 10^9 \times 5 \times 10^{-6} \times -7 \times 10^{-6}}{0.4^2}$

$= 2.0$ N attraction

26 $E = \frac{V}{d} = \frac{400}{0.038} = 1.05 \times 10^4\,\text{V}\,\text{m}^{-1}$

27 a A.

b B. There is a field in the BC direction from the left-hand current, and in the AB direction from the right-hand current.

c G. The field in directions A and C cancel.

28 $B = \frac{\mu_0 N}{L}$

$I = \frac{BL}{\mu_0 N}$

$= \frac{6.8 \times 10^{-6} \times 0.6}{1.257 \times 10^{-6} \times 50}$

$= 0.0649$

$= 64.9\,\text{mA}$

29 a The force is to the left, due to the magnetic induction in the soft iron.

b The force is more strongly to the left as the right end of the electromagnet is now a south pole.

c The force is to the right as the right end of the electromagnet is now a north pole.

30 $E = k\frac{q}{r^2} = 9 \times 10^9 \times \frac{30 \times 10^{-6}}{(0.3)^2}$

$= 3.0 \times 10^6\,\text{N}\,\text{C}^{-1}$

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EXTENDED RESPONSE

31 **a** $I = \frac{V}{R} = \frac{100}{50} = 2\,\text{A}$

$q = It$

$= (2.0\,\text{A})(1.0\,\text{s}) = 2.0\,\text{C}$

Number of electrons per second $= \frac{2.0}{1.60 \times 10^{-19}} = 1.3 \times 10^{19}$

b $W = qV = 1.60 \times 10^{-19} \times 100 = 1.60 \times 10^{-17}\,\text{J}$

c The electrical energy is converted into heat energy in the wire.

d $P = VI = 100 \times 2.0 = 2.0 \times 10^{2}\,\text{W}$

e There are 1.3×10^{19} electrons passing through the wire each second, each carrying energy of $1.60 \times 10^{-17}\,\text{J}$.

$\therefore$ Total energy per second $= 1.3 \times 10^{19} \times 1.60 \times 10^{-17} = 200\,\text{J}\,\text{s}^{-1} = 2.0 \times 10^{2}\,\text{W}$

f According to the answer to part d, power output of battery is $2.0 \times 10^{2}\,\text{W}$.

g These answers are the same because power is the energy given to each unit of charge (volt).

32 **a** The device is non-ohmic. The purpose of this device is to limit the current through a particular section of the circuit to a constant value regardless of the voltage across that part of the circuit.

b The resistance of the device increases with voltage. The current is constant over this range.

c $V_2 = (50\,\text{k}\Omega) \times (2.0\,\text{mA}) = 100\,\text{V}$

$V_1 = 250 - V_2$

Then $V_1 = 250 - 100 = 150\,\text{V}$

d $P = VI = (150\,\text{V}) \times (2.0\,\text{mA}) = 0.30\,\text{W}$

e $P = VI = (100\,\text{V}) \times (2.0\,\text{mA}) = 0.20\,\text{W}$

f $P = VI = (250\,\text{V}) \times (2.0\,\text{mA}) = 0.50\,\text{W}$

33 **a** Potential difference is the difference in electric potential between two points in a circuit. This drives the current within the circuit as the electrons move from high to low potential.

b $E = \frac{V}{d} = \frac{1000}{0.020} = 50\,000\,\text{V}\,\text{m}^{-1}$

c $\text{W} = \text{qEd} = 3 \times 1.602 \times 10^{-19} \times 50\,000 \times 0.020 = 4.806 \times 10^{-16}\,\text{J}$

d $\text{K} = \frac{1}{2}mv^2$

$\text{v} = \sqrt{\frac{2K}{m}} = \sqrt{\frac{2 \times 4.806 \times 10^{-16}}{3.27 \times 10^{-25}}}$

$= 5.42 \times 10^{4}\,\text{m}\,\text{s}^{-1}$

e $K = \frac{1}{2}mv^2 = \frac{1}{2} \times 3.27 \times 10^{-25} \times (6.5 \times 10^{4})^2 = 6.91 \times 10^{-16}\,\text{J}$

$W = qEd$

$E = \frac{W}{qd} = \frac{K}{qd} = \frac{6.91 \times 10^{-16}}{3 \times 1.602 \times 10^{-19} \times 0.020} = 71.867 \times 10^{3} = 71\,867\,\text{V}\,\text{m}^{-1}$

$E = \frac{V}{d}$

$V = Ed = 71\,867 \times 0.02 = 1437.4\,\text{V}$

f $\vec{E} = \frac{\vec{F}}{q}$

$F = Eq = 71\,867 \times 3 \times 1.602 \times 10^{-19}$

$= 3.45 \times 10^{-14}\,\text{N}$

34 **a** $V = IR$

$I = \frac{9}{5000} = 1.8\,\text{mA}$

b The sum of all potential differences needs to equal the supplied voltage (Kirchhoff's voltage law). So there will be a 4.5 V potential across each of the resistors.

c Bulbs 2 and 3 in parallel now have half the resistance of bulb 2 by itself (and therefore also bulb 1). So the voltage drop across bulb 1 must be twice the voltage drop across bulbs 2 and 3, i.e. 6 V across bulb 1 and 3 V across both bulbs 2 and 3.

Alternatively, the total resistance of the circuit is now 5000 + 2500 = 7500 kΩ, so the current is:

$I = \frac{9}{7500} = 1.2\,\text{mA}$

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$V = IR$, so:
$V_1 = 1.2 \times 10^{-3} \times 5000 = 6\,V$
$V_{2+3} = 1.2 \times 10^{-3} \times 2500 = 3\,V$

d $P = VI$
$P_1 = 6 \times 1.2 \times 10^{-3} = 7.2\,mW$
The current of 1.2 mA is split equally between P_2 and P_3, so:
$P_3 = 3 \times 0.6 \times 10^{-3} = 1.8\,mW$
$P_1 : P_3 = 7.2 : 1.8 = 4$
The first bulb has four times the amount of power as the third bulb.

35 a $E = k\frac{q}{r^2} = 9\times10^9 \times \frac{1.602\times10^{-19}}{(0.1)^2} = 14.4\,\mu N\,C^{-1}$

b $\vec{F} = k\frac{q_1q_2}{r^2} = 9\times10^9 \times \frac{1.602\times10^{-19}\times3\times1.602\times10^{-19}}{(0.02)^2} = 1.7 \times 10^{-24}\,N$

c $E = \frac{V}{d} = \frac{800}{0.020} = 40\,000\,V\,m^{-1}$

d $F = Eq = 40\,000 \times 3 \times 1.602 \times 10^{-19}$
$= 1.922 \times 10^{-14}\,N$

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